Consider the parametric equations:

$$x(t) = t^3 + t \qquad x(1) = 1^3 + 1 = 2$$

$$y(t) = -t^2 + 4t \qquad y(1) = -(1)^2 + 4(1) = 3$$

Compute $\frac{dx}{dt}$, $\frac{dy}{dt}$, and $\frac{dy}{dx}$ all in terms of t. Then find the line tangent to the above parametric equations at $t = 1$.

$(2,3)$

$$\frac{d}{dt}\; x = \underline{t^3 + t}_{\,dt}$$

$$y = -t^2 + 4t$$

$$\frac{dx}{dt} = 3t^2 + 1 \qquad \frac{dy}{dt} = -2t + 4$$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy \cdot \cancel{dt}}{dx \cdot \cancel{dt}} = \boxed{\frac{-2t+4}{3t^2+1}}$$

$$m = \frac{-2(1)+4}{3(1)^2+1} = \frac{2}{4} = \frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{1}{2}(x - 2)$$

Determine the area of the region below the parametric curve by the following set of parametric equations:

$$x = 4t^3 - t^2$$

$$y = t^4 + 2t^2$$

$$1 \le t \le 3$$

$$dx = (12t^2 - 2t)\,dt$$

$$A = \int_a^b \boxed{y}\,\boxed{dx}$$

$$A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t)\,dt$$

$$= \int_1^3 12t^6 - 2t^5 + 24t^4 - 4t^3\,dt$$

$$\left.\frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{\cancel{4}t^4}{\cancel{4}}\right|_1^3$$

$$\frac{12(3)^7}{7} - \frac{2(3)^6}{6} + \frac{24(3)^5}{5} - (3)^4 - \left[\frac{12(1)^7}{7} - \frac{2(1)^6}{6} + \frac{24(1)^5}{5} - (1)^4\right]$$

$$4591.54 - 5.18 = 4586.36$$

Find the exact length of the curve of $x = 1 + 3t^2, y = 4 + 2t^3, 0 \le t \le 1$.

$$\int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$x = 1 + 3t^2$

$\frac{dx}{dt} = 6t$

$y = 4 + 2t^3$

$\frac{dy}{dt} = 6t^2$

$$\int_0^1 \sqrt{(6t)^2 + (6t^2)^2}\, dt$$

$(6t)^2 = 6t \cdot 6t = 36t^2$

$$\int_0^1 \sqrt{36t^2 + 36t^4}\, dt$$

$$\int_0^1 \sqrt{\boxed{36t^2}(1+t^2)}\, dt = \int_0^1 6t\sqrt{1+t^2}\, dt$$

$U = 1 + t^2$

$du = 2t\,dt$

$\frac{du}{2t} = \frac{2t\,dt}{2t}$

$$\int_0^1 \frac{6\cancel{t}\sqrt{u}\, du}{2\cancel{t}}$$

$$\int_0^1 3\sqrt{u}\, du \rightarrow 3\int_0^1 \sqrt{u}\, du$$

$$3\int_0^1 u^{1/2}\, du = 3\left(\frac{u^{3/2}}{3/2}\right) \rightarrow \cancel{3}\left(\frac{2u^{3/2}}{\cancel{3}}\right) = 2u^{3/2}\Big|_0^1$$

$$2(1+t^2)^{3/2}\Big|_0^1 = 2\sqrt{8} - 2$$

$$= 4\sqrt{2} - 2$$