

Consider the parametric equations:

$$x(t) = t^3 + t \quad x(1) = 1^3 + 1 = 2$$

$$y(t) = -t^2 + 4t \quad y(1) = -(1)^2 + 4(1) = 3$$

Compute $\frac{dx}{dt}$, $\frac{dy}{dt}$, and $\frac{dy}{dx}$ all in terms of t . Then find the line tangent to the above parametric equations at $t = 1$.

(2, 3)

$$\frac{dx}{dt} = t^3 + t \quad y = -t^2 + 4t$$

$$\frac{d}{dt} x = \frac{d}{dt} (t^3 + t) \quad \frac{dy}{dt} = -2t + 4$$

$$\frac{d}{dt} x = 3t^2 + 1$$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{dy}{dt} \cdot dt}{\frac{dx}{dt} \cdot dt} = \frac{-2t + 4}{3t^2 + 1}$$

$$m = \frac{-2(1) + 4}{3(1)^2 + 1} = \frac{2}{4} = \frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{1}{2}(x - 2)$$

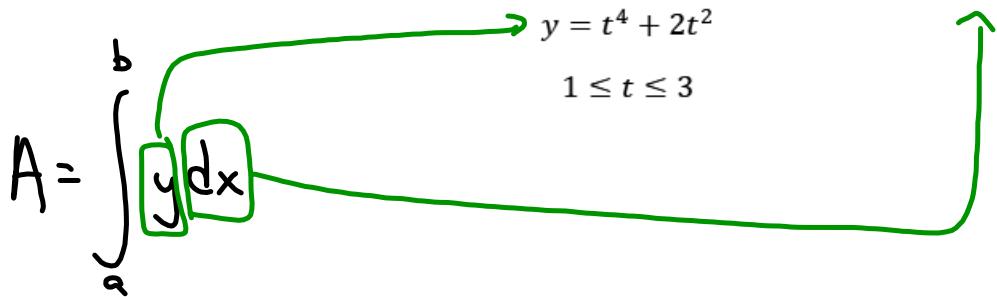
Determine the area of the region below the parametric curve by the following set of parametric equations:

$$x = 4t^3 - t^2$$

$$dx = (12t^2 - 2t)dt$$

$$y = t^4 + 2t^2$$

$$1 \leq t \leq 3$$



$$A = \int_{1}^{3} (t^4 + 2t^2)(12t^2 - 2t) dt$$

$$= \int_{1}^{3} 12t^6 - 2t^5 + 24t^4 - 4t^3 dt$$

$$\left. \frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{4t^4}{4} \right|_1^3$$

$$\frac{12(3)^7}{7} - \frac{2(3)^6}{6} + \frac{24(3)^5}{5} - (3)^4 - \left[\frac{12(1)^7}{7} - \frac{2(1)^6}{6} + \frac{24(1)^5}{5} - (1)^4 \right]$$

$$4591.54 - 5.18 = \boxed{4586.36}$$

Find the exact length of the curve of $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \leq t \leq 1$.

$$\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$
$$x = 1 + 3t^2$$
$$\frac{dx}{dt} = 6t$$
$$y = 4 + 2t^3$$
$$\frac{dy}{dt} = 6t^2$$
$$(6t)^2 = 6t \cdot 6t = 36t^2$$
$$\int_0^1 \sqrt{36t^2 + 36t^4} dt$$
$$\int_0^1 \sqrt{36t^2(1+t^2)} dt = \int_0^1 6t\sqrt{1+t^2} dt$$
$$U = 1+t^2$$
$$\frac{du}{dt} = 2t \Rightarrow dt = \frac{du}{2t}$$
$$\int_0^1 \frac{6t\sqrt{u} du}{2t} = \int_0^1 3\sqrt{u} du$$
$$3 \int_0^1 u^{1/2} du = 3 \left(\frac{u^{3/2}}{3/2} \right) \Big|_0^1 = 3 \left(\frac{2u^{3/2}}{3} \right) \Big|_0^1 = 2u^{3/2} \Big|_0^1$$
$$2(1+t^2) \Big|_0^1 = 2\sqrt{8} - 2$$
$$= 4\sqrt{2} - 2$$