

Derivative Quiz Review

For each problem, find the average rate of change of the function over the given interval and also find the instantaneous rate of change at the leftmost value of the given interval.

1) $y = -\frac{1}{x-2}; [0, \frac{1}{2}]$

2) $y = -x^2 - 2x - 1; [0, \frac{1}{2}]$

3) $y = -x^2 - x + 1; [0, \frac{1}{2}]$

4) $y = 2x^2 + 2x - 2; [1, \frac{3}{2}]$

Differentiate each function with respect to x .

5) $f(x) = -\frac{1}{3}x^4$

6) $f(x) = 5x^2$

7) $f(x) = -\frac{3}{2}x^{-2}$

8) $f(x) = x^{-3}\sqrt{2}$

9) $f(x) = \frac{\sqrt{5}}{x^2}$

10) $f(x) = -\frac{5}{3} - 2x^{-3} + 5x^{-5}$

11) $f(x) = -\frac{1}{2}x^{\frac{3}{2}} - 1 - \frac{1}{4x^5}$

12) $f(x) = -\frac{5}{3}x^2 + 4x^{\frac{4}{5}} + \frac{2}{5}$

13) $f(x) = \frac{\sqrt{3}}{x}$

14) $f(x) = \frac{\sqrt{5}}{x^3} - \frac{1}{4x^5}$

15) $f(x) = 2\sqrt[5]{x}(4x^4 - 3)$

16) $f(x) = (4 - 2x^{-2})(-3x^5 + 2)$

17) $f(x) = (-4\sqrt[3]{x^2} + 4)(-3x^2 + 3)$

18) $f(x) = (-5\sqrt[5]{x} - 3) \cdot -4x^2$

19) $f(x) = \left(4x^{\frac{2}{3}} - 3\right) \cdot -2x^3$

20) $f(x) = \left(2 + \frac{2}{x^4}\right)(-x^4 - 2)$

21) $f(x) = (5 - 4x^{-2})(-2x^3 + 2x^2 + 1)$

22) $f(x) = \left(-2 - \frac{2}{x^2}\right)(4x^4 + 4x^2 + 1)$

23) $f(x) = (3 + 2x^{-5})(-3x^3 + 3x^2 - 1)$

$$24) f(x) = (-x^3 - \sqrt[4]{x} - 1)(-5x^4 - 3)$$

$$25) f(x) = \frac{4x^2 + 4}{3\sqrt[3]{x} + 3}$$

$$26) f(x) = \frac{1}{4 - \frac{2}{x^3}}$$

$$27) f(x) = \frac{1}{\sqrt[4]{x} - 2}$$

$$28) f(x) = \frac{3x^5 + x^2}{3 + x^{-2}}$$

$$29) f(x) = \frac{2x^5 - 3x^3 + 4}{2 - 5x^{-4}}$$

$$30) f(x) = \frac{x^5 - 2x^4 - 5x^3}{5 - \frac{1}{x^3}}$$

$$31) f(x) = \frac{4x^5 + 2x^4 - x^2}{3 - 3x^{-4}}$$

$$32) f(x) = \frac{3x^4 - 3x^3}{2\sqrt[5]{x^2} + 5}$$

$$33) f(x) = \frac{3x^5 + 2}{2 + x^{-4}}$$

$$34) f(x) = \frac{5x^5 + 3}{2 - \frac{2}{x^5}}$$

For each problem, find the indicated derivative with respect to x .

$$35) f(x) = -2\sqrt[5]{x} + \frac{1}{x^3} \quad \text{Find } f'''$$

$$36) f(x) = -2x^5 \quad \text{Find } f''$$

$$37) f(x) = 5x \quad \text{Find } f'''$$

$$38) f(x) = x^{\frac{5}{2}} + 5x^{\frac{1}{4}} + 4x^{-2} \quad \text{Find } f''$$

$$39) f(x) = 2\sqrt[4]{x} + x^{-3} \quad \text{Find } f'''$$

$$40) f(x) = 4x^{\frac{4}{5}} - \frac{3}{x} \quad \text{Find } f^{(4)}$$

$$41) f(x) = -3x^5 - 4x^{\frac{2}{5}} - 3x^{-2} \quad \text{Find } f^{(4)}$$

$$42) f(x) = 4x^4 + 3x^{-2} + \frac{5}{x^4} \quad \text{Find } f''$$

$$43) f(x) = 5x^3 + 3\sqrt[3]{x^2} \quad \text{Find } f^{(4)}$$

$$44) f(x) = -x^3 + 2x^{\frac{5}{2}} \quad \text{Find } f'''$$

Answers to Derivative Quiz Review

1) Average: $\frac{1}{3}$ Instant.: $\frac{1}{4}$

2) Average: $-\frac{5}{2}$ Instant.: -2

3) Average: $-\frac{3}{2}$ Instant.: -1

4) Average: 7 Instant.: 6

5) $f'(x) = -\frac{4x^3}{3}$

6) $f'(x) = 10x$

7) $f'(x) = \frac{3}{x^3}$

8) $f'(x) = -\frac{3\sqrt{2}}{x^4}$

9) $f'(x) = -\frac{2\sqrt{5}}{x^3}$

10) $f'(x) = \frac{6}{x^4} - \frac{25}{x^6}$

11) $f'(x) = -\frac{3x^{\frac{1}{2}}}{4} + \frac{5}{4x^6}$

12) $f'(x) = -\frac{10x}{3} + \frac{16}{5x^5}$

13) $f'(x) = -\frac{\sqrt{3}}{x^2}$

14) $f'(x) = -\frac{3\sqrt{5}}{x^4} + \frac{5}{4x^6}$

15) $f'(x) = 2x^{\frac{1}{5}} \cdot 16x^3 + (4x^4 - 3) \cdot \frac{2}{5}x^{-\frac{4}{5}}$

$$= \frac{168x^{\frac{16}{5}}}{5} - \frac{6}{5x^{\frac{4}{5}}}$$

16) $f'(x) = (4 - 2x^{-2}) \cdot -15x^4 + (-3x^5 + 2) \cdot 4x^{-3}$
 $= -60x^4 + 18x^2 + \frac{8}{x^3}$

17) $f'(x) = (-4x^{\frac{2}{3}} + 4) \cdot -6x + (-3x^2 + 3) \cdot -\frac{8}{3}x^{-\frac{1}{3}}$
 $= 32x^{\frac{5}{3}} - 24x - \frac{8}{x^{\frac{1}{3}}}$

18) $f'(x) = (-5x^{\frac{1}{5}} - 3) \cdot -8x - 4x^2 \cdot -x^{-\frac{4}{5}}$
 $= 44x^{\frac{6}{5}} + 24x$

19) $f'(x) = (4x^{\frac{2}{3}} - 3) \cdot -6x^2 - 2x^3 \cdot \frac{8}{3}x^{-\frac{1}{3}}$
 $= -\frac{88x^{\frac{8}{3}}}{3} + 18x^2$

20) $f'(x) = (2 + 2x^{-4}) \cdot -4x^3 + (-x^4 - 2) \cdot -8x^{-5}$
 $= -8x^3 + \frac{16}{x^5}$

21) $f'(x) = (5 - 4x^{-2})(-6x^2 + 4x) + (-2x^3 + 2x^2 + 1) \cdot 8x^{-3}$
 $= -30x^2 + 20x + 8 + \frac{8}{x^3}$

22) $f'(x) = (-2 - 2x^{-2})(16x^3 + 8x) + (4x^4 + 4x^2 + 1) \cdot 4x^{-3}$
 $= -32x^3 - 32x + \frac{4}{x^3}$

23) $f'(x) = (3 + 2x^{-5})(-9x^2 + 6x) + (-3x^3 + 3x^2 - 1) \cdot -10x^{-6}$
 $= -27x^2 + 18x + \frac{12}{x^3} - \frac{18}{x^4} + \frac{10}{x^6}$

24) $f'(x) = (-x^3 - x^{\frac{1}{4}} - 1) \cdot -20x^3 + (-5x^4 - 3) \left(-3x^2 - \frac{1}{4}x^{-\frac{3}{4}}\right)$
 $= 35x^6 + \frac{85x^{\frac{5}{4}}}{4} + 20x^3 + 9x^2 + \frac{3}{4x^{\frac{3}{4}}}$

$$25) f'(x) = \frac{\left(3x^{\frac{1}{3}} + 3\right) \cdot 8x - (4x^2 + 4) \cdot x^{-\frac{2}{3}}}{\left(3x^{\frac{1}{3}} + 3\right)^2}$$

$$= \frac{20x^2 + 24x^{\frac{5}{3}} - 4}{9x^3 + 18x + 9x^{\frac{2}{3}}}$$

$$26) f'(x) = -\frac{6x^{-4}}{(4 - 2x^{-3})^2}$$

$$= -\frac{3x^2}{8x^6 - 8x^3 + 2}$$

$$27) f'(x) = -\frac{\frac{1}{4}x^{-\frac{3}{4}}}{\left(x^4 - 2\right)^2}$$

$$= -\frac{1}{4x^4 - 16x + 16x^{\frac{3}{4}}}$$

$$28) f'(x) = \frac{(3 + x^{-2})(15x^4 + 2x) - (3x^5 + x^2) \cdot -2x^{-3}}{(3 + x^{-2})^2}$$

$$= \frac{45x^8 + 21x^6 + 6x^5 + 4x^3}{9x^4 + 6x^2 + 1}$$

$$29) f'(x) = \frac{(2 - 5x^{-4})(10x^4 - 9x^2) - (2x^5 - 3x^3 + 4) \cdot 20x^{-5}}{(2 - 5x^{-4})^2}$$

$$= \frac{20x^{12} - 18x^{10} - 90x^8 + 105x^6 - 80x^3}{4x^8 - 20x^4 + 25}$$

$$30) f'(x) = \frac{(5 - x^{-3})(5x^4 - 8x^3 - 15x^2) - (x^5 - 2x^4 - 5x^3) \cdot 3x^{-4}}{(5 - x^{-3})^2}$$

$$= \frac{25x^{10} - 40x^9 - 75x^8 - 8x^7 + 14x^6 + 30x^5}{25x^6 - 10x^3 + 1}$$

$$31) f'(x) = \frac{(3 - 3x^{-4})(20x^4 + 8x^3 - 2x) - (4x^5 + 2x^4 - x^2) \cdot 12x^{-5}}{(3 - 3x^{-4})^2}$$

$$= \frac{20x^{12} + 8x^{11} - 2x^9 - 36x^8 - 16x^7 + 6x^5}{3x^8 - 6x^4 + 3}$$

$$32) f'(x) = \frac{\left(2x^{\frac{2}{5}} + 5\right)(12x^3 - 9x^2) - (3x^4 - 3x^3) \cdot \frac{4}{5}x^{-\frac{3}{5}}}{\left(2x^{\frac{2}{5}} + 5\right)^2}$$

$$= \frac{108x^{\frac{17}{5}} + 300x^3 - 78x^{\frac{12}{5}} - 225x^2}{20x^{\frac{4}{5}} + 100x^{\frac{2}{5}} + 125}$$

$$33) f'(x) = \frac{(2 + x^{-4}) \cdot 15x^4 - (3x^5 + 2) \cdot -4x^{-5}}{(2 + x^{-4})^2}$$

$$= \frac{30x^{12} + 27x^8 + 8x^3}{4x^8 + 4x^4 + 1}$$

$$34) f'(x) = \frac{(2 - 2x^{-5}) \cdot 25x^4 - (5x^5 + 3) \cdot 10x^{-6}}{(2 - 2x^{-5})^2}$$

$$= \frac{25x^{14} - 50x^9 - 15x^4}{2x^{10} - 4x^5 + 2}$$

$$35) f'''(x) = -\frac{72}{125x^{\frac{14}{5}}} - \frac{60}{x^6}$$

$$36) f'''(x) = -40x^3$$

$$37) f'''(x) = 0$$

$$38) f''(x) = \frac{15x^{\frac{1}{2}}}{4} - \frac{15}{16x^{\frac{7}{4}}} + \frac{24}{x^4}$$

$$39) f'''(x) = \frac{21}{32x^{\frac{11}{4}}} - \frac{60}{x^6}$$

$$40) f^{(4)}(x) = -\frac{1056}{625x^{\frac{16}{5}}} - \frac{72}{x^5}$$

$$41) f^{(4)}(x) = -360x + \frac{2496}{625x^{\frac{18}{5}}} - \frac{360}{x^6}$$

$$42) f''(x) = 48x^2 + \frac{18}{x^4} + \frac{100}{x^6}$$

$$43) f^{(4)}(x) = -\frac{56}{27x^{\frac{10}{3}}}$$

$$44) f'''(x) = -6 + \frac{15}{4x^{\frac{1}{2}}}$$