Calculus
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Derivative Review

Differentiate each function with respect to *x*.

1) $f(x) = \frac{5}{2}x^5 + 2x^4 + \frac{2}{5}x^{\frac{1}{2}}$

2) $y = -\frac{4}{5}x^2 + 2x^{\frac{1}{3}} + \frac{2}{5}x^{-4}$

3) $f(x) = \frac{3}{5}x^3 - \frac{3}{4}x - 5x^{\frac{1}{3}}$

4) $f(x) = \frac{1}{2}x^{\frac{4}{5}} + 2 + \frac{1}{4}x^{-3}$

5) $y = \left(-5x^{\frac{2}{3}} - 5\right)(2x^2 + 1)$

6) $y = (-4 - 4x^{-5})(-x^2 - 3)$

7) $f(x) = (1 - 2x^{-2})(-5x^3 - 1)$

8) $f(x) = \left(-4 - \frac{1}{x^4}\right)(-4x^4 - 3)$

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9) $f(x) = \dfrac{3x^3 + 1}{5 - \dfrac{2}{x^4}}$

10) $f(x) = \dfrac{x^2 - 5}{5\sqrt[5]{x} + 2}$

11) $y = \dfrac{x^4 + 4x^2}{3 + \dfrac{5}{x^2}}$

12) $y = \dfrac{x^2 - 5}{2\sqrt[4]{x} + 2}$

13) $y = (x^5 + 2)^{-3}$

14) $y = (4x^5 - 5)^3$

15) $f(x) = (5x^5 - 1)^{-5}$

16) $f(x) = (2x^4 - 5)^{\frac{1}{3}}$

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17) $f(x) = \frac{(5x + 3)^{-3}}{(-3x^2 - 5)^{\frac{1}{4}}}$

18) $f(x) = \frac{(-5x^4 - 3)^{\frac{1}{3}}}{(4x^5 - 1)^2}$

19) $y = (4x^3 + 5)^{-5} \cdot (x^5 + 3)^{\frac{1}{4}}$

20) $f(x) = (5x^5 - 4)^{\frac{1}{3}} \cdot (-4x^3 - 1)^5$

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For each problem, find the indicated derivative with respect to *x*.

21) $y = 3x^4$ Find $\frac{d^2y}{dx^2}$

22) $f(x) = -4x^2 - 2x$ Find $f^{(4)}$

23) $f(x) = -5x^5 + 5x^4 - 2x^3$ Find f'''

24) $f(x) = x^4$ Find f''

Differentiate each function with respect to *x*.

25) $f(x) = \sin 3x^3$

26) $y = \sin x^5$

27) $f(x) = \frac{-x^3 + 2}{\sin 3x^5}$

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28) $f(x) = \sin \frac{x^5}{3x^2 + 4}$

For each problem, find the derivative of the function at the given value.

29) $y = -x^3 + 2x^2 + 2$ at $x = -1$

30) $y = -2\csc(x)$ at $x = -\frac{\pi}{4}$

31) $y = \frac{x^2}{2} - 3x + \frac{13}{2}$ at $x = 0$

32) $y = \frac{16x}{x^2 + 16}$ at $x = 1$

33) $y = \tan(2x)$ at $x = -\frac{\pi}{2}$

34) $y = \frac{1}{x^2 - 9}$ at $x = 0$

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Solve each optimization problem.

35) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 108 ft^3 of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

36) Two vertical poles, one 12 ft high and the other 24 ft high, stand 15 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?

37) A farmer wants to construct a rectangular pigpen using 300 ft of fencing. The pen will be built next to an existing stone wall, so only three sides of fencing need to be constructed to enclose the pen. What dimensions should the farmer use to construct the pen with the largest possible area?

38) A geometry student wants to draw a rectangle inscribed in a semicircle of radius 8. If one side must be on the semicircle's diameter, what is the area of the largest rectangle that the student can draw?

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39) Which point on the graph of $y = \sqrt{x}$ is closest to the point $(4, 0)$?

40) A rancher wants to construct two identical rectangular corrals using 100 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the position, velocity, and acceleration at the given value for t.

41) $s(t) = -t^4 + 9t^3$; at $t = 5$

42) $s(t) = -t^3 + 13t^2 - 40t$; at $t = 5$

43) $s(t) = t^4 - 12t^3$; at $t = 3$

44) $s(t) = t^2 - 10t - 39$; at $t = 8$

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Solve each related rate problem.

45) A hypothetical square grows so that the length of its sides are increasing at a rate of 4 m/min. How fast is the area of the square increasing when the sides are 12 m each?

46) A hypothetical square grows so that the length of its diagonals are increasing at a rate of 6 m/min. How fast is the area of the square increasing when the diagonals are 13 m each?

47) A crowd gathers around a movie star, forming a circle. The radius of the crowd increases at a rate of 4 ft/sec. How fast is the area taken up by the crowd increasing when the radius is 14 ft?

48) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The radius of the spill increases at a rate of 6 m/min. How fast is the area of the spill increasing when the radius is 10 m?

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Answers to Derivative Review

1) $f'(x) = \frac{25}{2}x^4 + 8x^3 + \frac{1}{5}x^{-\frac{1}{2}}$
$= \frac{25x^4}{2} + 8x^3 + \frac{1}{5x^{\frac{1}{2}}}$

2) $\frac{dy}{dx} = -\frac{8}{5}x + \frac{2}{3}x^{-\frac{2}{3}} - \frac{8}{5}x^{-5}$
$= -\frac{8x}{5} + \frac{2}{3x^{\frac{2}{3}}} - \frac{8}{5x^5}$

3) $f'(x) = \frac{9}{5}x^2 - \frac{3}{4} - \frac{5}{3}x^{-\frac{2}{3}}$
$= \frac{9x^2}{5} - \frac{3}{4} - \frac{5}{3x^{\frac{2}{3}}}$

4) $f'(x) = \frac{2}{5}x^{-\frac{1}{5}} - \frac{3}{4}x^{-4}$
$= \frac{2}{5x^{\frac{1}{5}}} - \frac{3}{4x^4}$

5) $\frac{dy}{dx} = \left(-5x^{\frac{2}{3}} - 5\right) \cdot 4x + \left(2x^2 + 1\right) \cdot -\frac{10}{3}x^{-\frac{1}{3}}$
$= -\frac{80x^{\frac{5}{3}}}{3} - 20x - \frac{10}{3x^{\frac{1}{3}}}$

6) $\frac{dy}{dx} = \left(-4 - 4x^{-5}\right) \cdot -2x + \left(-x^2 - 3\right) \cdot 20x^{-6}$
$= 8x - \frac{12}{x^4} - \frac{60}{x^6}$

7) $f'(x) = \left(1 - 2x^{-2}\right) \cdot -15x^2 + \left(-5x^3 - 1\right) \cdot 4x^{-3}$
$= -15x^2 + 10 - \frac{4}{x^3}$

8) $f'(x) = \left(-4 - x^{-4}\right) \cdot -16x^3 + \left(-4x^4 - 3\right) \cdot 4x^{-5}$
$= 64x^3 - \frac{12}{x^5}$

9) $f'(x) = \frac{\left(5 - 2x^{-4}\right) \cdot 9x^2 - \left(3x^3 + 1\right) \cdot 8x^{-5}}{\left(5 - 2x^{-4}\right)^2}$
$= \frac{45x^{10} - 42x^6 - 8x^3}{25x^8 - 20x^4 + 4}$

10) $f'(x) = \frac{\left(5x^{\frac{1}{5}} + 2\right) \cdot 2x - \left(x^2 - 5\right) \cdot x^{-\frac{4}{5}}}{\left(5x^{\frac{1}{5}} + 2\right)^2}$
$= \frac{9x^2 + 4x^{\frac{9}{5}} + 5}{25x^{\frac{6}{5}} + 20x + 4x^{\frac{4}{5}}}$

11) $\frac{dy}{dx} = \frac{\left(3 + 5x^{-2}\right)\left(4x^3 + 8x\right) - \left(x^4 + 4x^2\right) \cdot -10x^{-3}}{\left(3 + 5x^{-2}\right)^2}$
$= \frac{12x^7 + 54x^5 + 80x^3}{9x^4 + 30x^2 + 25}$

12) $\frac{dy}{dx} = \frac{\left(2x^{\frac{1}{4}} + 2\right) \cdot 2x - \left(x^2 - 5\right) \cdot \frac{1}{2}x^{-\frac{3}{4}}}{\left(2x^{\frac{1}{4}} + 2\right)^2}$
$= \frac{7x^2 + 8x^{\frac{7}{4}} + 5}{8x^{\frac{5}{4}} + 16x + 8x^{\frac{3}{4}}}$

13) $\frac{dy}{dx} = -3\left(x^5 + 2\right)^{-4} \cdot 5x^4$
$= -\frac{15x^4}{\left(x^5 + 2\right)^4}$

14) $\frac{dy}{dx} = 3\left(4x^5 - 5\right)^2 \cdot 20x^4$
$= 60x^4\left(4x^5 - 5\right)^2$

15) $f'(x) = -5\left(5x^5 - 1\right)^{-6} \cdot 25x^4$
$= -\frac{125x^4}{\left(5x^5 - 1\right)^6}$

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16) $f'(x) = \frac{1}{3}(2x^4 - 5)^{-\frac{2}{3}} \cdot 8x^3$

$= \frac{8x^3}{3(2x^4 - 5)^{\frac{2}{3}}}$

17) $f'(x) = \frac{(-3x^2 - 5)^{\frac{1}{4}} \cdot -3(5x + 3)^{-4} \cdot 5 - (5x + 3)^{-3} \cdot \frac{1}{4}(-3x^2 - 5)^{-\frac{3}{4}} \cdot -6x}{\left((-3x^2 - 5)^{\frac{1}{4}}\right)^2}$

$= \frac{3(35x^2 + 50 + 3x)}{2(-3x^2 - 5)^{\frac{5}{4}} \cdot (5x + 3)^4}$

18) $f'(x) = \frac{(4x^5 - 1)^2 \cdot \frac{1}{3}(-5x^4 - 3)^{-\frac{2}{3}} \cdot -20x^3 - (-5x^4 - 3)^{\frac{1}{3}} \cdot 2(4x^5 - 1) \cdot 20x^4}{((4x^5 - 1)^2)^2}$

$= \frac{20x^3(26x^5 + 18x + 1)}{3(-5x^4 - 3)^{\frac{2}{3}} \cdot (4x^5 - 1)^3}$

19) $\frac{dy}{dx} = (4x^3 + 5)^{-5} \cdot \frac{1}{4}(x^5 + 3)^{-\frac{3}{4}} \cdot 5x^4 + (x^5 + 3)^{\frac{1}{4}} \cdot -5(4x^3 + 5)^{-6} \cdot 12x^2$

$= \frac{5x^2(-44x^5 + 5x^2 - 144)}{4(x^5 + 3)^{\frac{3}{4}} \cdot (4x^3 + 5)^6}$

20) $f'(x) = (5x^5 - 4)^{\frac{1}{3}} \cdot 5(-4x^3 - 1)^4 \cdot -12x^2 + (-4x^3 - 1)^5 \cdot \frac{1}{3}(5x^5 - 4)^{-\frac{2}{3}} \cdot 25x^4$

$= \frac{5x^2(-4x^3 - 1)^4(-200x^5 + 144 - 5x^2)}{3(5x^5 - 4)^{\frac{2}{3}}}$

21) $\frac{d^2y}{dx^2} = 36x^2$

22) $f^{(4)}(x) = 0$

23) $f'''(x) = -300x^2 + 120x - 12$

24) $f''(x) = 12x^2$

25) $f'(x) = \cos 3x^3 \cdot 9x^2$
$= 9x^2 \cos 3x^3$

26) $\frac{dy}{dx} = \cos x^5 \cdot 5x^4$
$= 5x^4 \cos x^5$

27) $f'(x) = \frac{\sin 3x^5 \cdot -3x^2 - (-x^3 + 2) \cdot \cos 3x^5 \cdot 15x^4}{\sin^2 3x^5}$

$= \frac{3x^2(-\sin 3x^5 + 5x^5 \cos 3x^5 - 10x^2 \cos 3x^5)}{\sin^2 3x^5}$

28) $f'(x) = \cos \frac{x^5}{3x^2 + 4} \cdot \frac{(3x^2 + 4) \cdot 5x^4 - x^5 \cdot 6x}{(3x^2 + 4)^2}$

$= \frac{x^4 \cos \frac{x^5}{3x^2 + 4} \cdot (9x^2 + 20)}{(3x^2 + 4)^2}$

29) $\left.\frac{dy}{dx}\right|_{x = -1} = -7$

30) $\left.\frac{dy}{dx}\right|_{x = -\frac{\pi}{4}} = 2\sqrt{2}$

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31) $\left.\frac{dy}{dx}\right|_{x=0} = -3$ 32) $\left.\frac{dy}{dx}\right|_{x=1} = \frac{240}{289}$ 33) $\left.\frac{dy}{dx}\right|_{x=-\frac{\pi}{2}} = 2$ 34) $\left.\frac{dy}{dx}\right|_{x=0} = 0$

35) A = the area of the glass x = the length of the sides of the square bottom

Function to minimize: $A = x^2 + 4x \cdot \frac{108}{x^2}$ where $0 < x < \infty$

Dimensions of the aquarium: 6 ft by 6 ft by 3 ft tall

36) L = the total length of rope x = the horizontal distance from the short pole to the stake

Function to minimize: $L = \sqrt{x^2 + 12^2} + \sqrt{(15 - x)^2 + 24^2}$ where $0 \le x \le 15$

Stake should be placed: 5 ft from the short pole (or 10 ft from the long pole)

37) A = the area of the pigpen x = the length of the sides perpendicular to the stone wall

Function to maximize: $A = x(300 - 2x)$ where $0 < x < 150$

Dimensions of the pigpen: 75 ft (perpendicular to wall) by 150 ft (parallel to wall)

38) A = the area of the rectangle x = half the base of the rectangle

Function to maximize: $A = 2x\sqrt{8^2 - x^2}$ where $0 < x < 8$

Area of largest rectangle: 64

39) d = the distance from point $(4, 0)$ to a point on the curve x = the x-coordinate of a point on the curve

Function to minimize: $d = \sqrt{(x - 4)^2 + (\sqrt{x})^2}$ where $-\infty < x < \infty$

Point on the curve that is closest to the point $(4, 0)$: $\left(\frac{7}{2}, \frac{\sqrt{14}}{2}\right)$

40) A = the total area of the two corrals x = the length of the non-adjacent sides of each corral

Function to maximize: $A = 2x \cdot \frac{100 - 4x}{3}$ where $0 < x < 25$

Dimensions of each corall: $\frac{25}{2}$ ft (non-adjecent sides) by $\frac{50}{3}$ ft (adjacent sides)

41) $s(5) = 500, v(5) = 175, a(5) = -30$

42) $s(5) = 0, v(5) = 15, a(5) = -4$

43) $s(3) = -243, v(3) = -216, a(3) = -108$

44) $s(8) = -55, v(8) = 6, a(8) = 2$

45) A = area of square s = length of sides t = time

Equation: $A = s^2$ Given rate: $\frac{ds}{dt} = 4$ Find: $\left.\frac{dA}{dt}\right|_{s=12}$

$\left.\frac{dA}{dt}\right|_{s=12} = 2s \cdot \frac{ds}{dt} = 96$ m²/min

46) A = area of square x = length of diagonals t = time

Equation: $A = \frac{x^2}{2}$ Given rate: $\frac{dx}{dt} = 6$ Find: $\left.\frac{dA}{dt}\right|_{x=13}$

$\left.\frac{dA}{dt}\right|_{x=13} = x \cdot \frac{dx}{dt} = 78$ m²/min

47) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dr}{dt} = 4$ Find: $\left.\frac{dA}{dt}\right|_{r=14}$

$\left.\frac{dA}{dt}\right|_{r=14} = 2\pi r \cdot \frac{dr}{dt} = 112\pi$ ft²/sec

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48) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dr}{dt} = 6$ Find: $\left.\frac{dA}{dt}\right|_{r = 10}$

$$\left.\frac{dA}{dt}\right|_{r = 10} = 2\pi r \cdot \frac{dr}{dt} = 120\pi \text{ m}^2\text{/min}$$

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