

Differentiation

For each problem, find the average rate of change of the function over the given interval.

1) $y = x^2 - 2$; $[0, 3]$

2) $y = 2x^2 + 1$; $[0, 1]$

3) $y = 2x^2 + 2$; $[0, 1]$

4) $y = 2x^2 - 1$; $[-1, 2]$

For each problem, find the instantaneous rate of change of the function at the given value.

5) $y = -2x^2 - 1$; -1

6) $y = x^2 + 2x + 2$; -1

7) $y = -x^2 - x - 1$; -1

8) $y = 2x^2 + 1$; -1

Differentiate each function with respect to x .

9) $f(x) = 5x^5$

10) $f(x) = \frac{4}{3}x^3$

11) $f(x) = -3x^2 + 3x$

12) $f(x) = -\frac{3}{5}x$

13) $f(x) = -3x^5 - \frac{5}{2}x^3 - 4x$

14) $f(x) = \frac{1}{3}x^4 + \frac{1}{3}x^2$

15) $f(x) = 5x^2(3x^4 + 3)$

16) $f(x) = (5x^5 + 2)(-5x^2 + 5)$

17) $f(x) = (2x^3 - 2)(-2x^5 - x^3 + 2)$

$$18) f(x) = (-5x^5 + 2)(4x^4 + 2x^3 - 3)$$

$$19) f(x) = \frac{4x^3}{2x^4 + 3}$$

$$20) f(x) = \frac{x^3}{2x^4 + 5}$$

$$21) f(x) = \frac{3x^4 + x^3 + 2x^2}{4x^2 + 3}$$

$$22) f(x) = \frac{4}{x^2 + 3}$$

$$23) f(x) = ((-5x^3 - 4)^3 + 4)^2$$

$$24) f(x) = (-3x^2 + 2)^3 \cdot (-4x^5 - 3)^5$$

$$25) f(x) = (-5x^5 - 3)^3 \cdot (-x^4 + 2)^5$$

$$26) f(x) = (-5x^4 - 1)^3$$

$$27) f(x) = \frac{(-3x^3 - 1)^3}{(-x^4 + 3)^5}$$

$$28) f(x) = ((2x^5 - 5)^3 + 3)^4$$

$$29) f(x) = \tan(\sin x^4)$$

$$30) f(x) = \csc 3x^2$$

$$31) f(x) = \cot x^4$$

32) $f(x) = \cot 2x^2$

For each problem, find the indicated derivative with respect to x .

33) $f(x) = -4x^5 + 5x^2$ Find $f^{(4)}$

34) $f(x) = 4x^4 + 4x^2 - x$ Find f'''

35) $f(x) = -5x^5 + x^4 + x^2$ Find f'''

36) $f(x) = x$ Find f'''

For each problem, find the slope of the function at the given value.

37) $f(x) = \frac{2}{x^2 + 1}$ at $x = 2$

38) $f(x) = \frac{x^2}{2} - 2x + 1$ at $x = 1$

39) $f(x) = -(-3x + 3)^{\frac{1}{2}}$ at $x = -1$

40) $f(x) = -x^2 + 2x - 2$ at $x = 0$

Answers to Differentiation

1) 3	2) 2	3) 2	4) 2
5) 4	6) 0	7) 1	8) -4
9) $f'(x) = 25x^4$	10) $f'(x) = 4x^2$	11) $f'(x) = -6x + 3$	12) $f'(x) = -\frac{3}{5}$

$$13) f'(x) = -15x^4 - \frac{15x^2}{2} - 4 \quad 14) f'(x) = \frac{4x^3}{3} + \frac{2x}{3}$$

$$15) f'(x) = 5x^2 \cdot 12x^3 + (3x^4 + 3) \cdot 10x = 90x^5 + 30x \quad 16) f'(x) = (5x^5 + 2) \cdot -10x + (-5x^2 + 5) \cdot 25x^4 = -175x^6 + 125x^4 - 20x$$

$$17) f'(x) = (2x^3 - 2)(-10x^4 - 3x^2) + (-2x^5 - x^3 + 2) \cdot 6x^2 = -32x^7 - 12x^5 + 20x^4 + 18x^2$$

$$18) f'(x) = (-5x^5 + 2)(16x^3 + 6x^2) + (4x^4 + 2x^3 - 3) \cdot -25x^4 = -180x^8 - 80x^7 + 75x^4 + 32x^3 + 12x^2$$

$$19) f'(x) = \frac{(2x^4 + 3) \cdot 12x^2 - 4x^3 \cdot 8x^3}{(2x^4 + 3)^2} = \frac{-8x^6 + 36x^2}{4x^8 + 12x^4 + 9} \quad 20) f'(x) = \frac{(2x^4 + 5) \cdot 3x^2 - x^3 \cdot 8x^3}{(2x^4 + 5)^2} = \frac{-2x^6 + 15x^2}{4x^8 + 20x^4 + 25}$$

$$21) f'(x) = \frac{(4x^2 + 3)(12x^3 + 3x^2 + 4x) - (3x^4 + x^3 + 2x^2) \cdot 8x}{(4x^2 + 3)^2} = \frac{24x^5 + 4x^4 + 36x^3 + 9x^2 + 12x}{16x^4 + 24x^2 + 9}$$

$$22) f'(x) = -\frac{4 \cdot 2x}{(x^2 + 3)^2} = -\frac{8x}{x^4 + 6x^2 + 9}$$

$$23) f'(x) = 2((-5x^3 - 4)^3 + 4) \cdot 3(-5x^3 - 4)^2 \cdot -15x^2 = -90x^2(-5x^3 - 4)^2((-5x^3 - 4)^3 + 4)$$

$$24) f'(x) = (-3x^2 + 2)^3 \cdot 5(-4x^5 - 3)^4 \cdot -20x^4 + (-4x^5 - 3)^5 \cdot 3(-3x^2 + 2)^2 \cdot -6x = 2x(-3x^2 + 2)^2 \cdot (-4x^5 - 3)^4(186x^5 - 100x^3 + 27)$$

$$25) f'(x) = (-5x^5 - 3)^3 \cdot 5(-x^4 + 2)^4 \cdot -4x^3 + (-x^4 + 2)^5 \cdot 3(-5x^5 - 3)^2 \cdot -25x^4 = 5x^3(-5x^5 - 3)^2 \cdot (-x^4 + 2)^4(35x^5 + 12 - 30x)$$

$$26) f'(x) = 3(-5x^4 - 1)^2 \cdot -20x^3 = -60x^3(-5x^4 - 1)^2$$

$$27) f'(x) = \frac{(-x^4 + 3)^5 \cdot 3(-3x^3 - 1)^2 \cdot -9x^2 - (-3x^3 - 1)^3 \cdot 5(-x^4 + 3)^4 \cdot -4x^3}{((-x^4 + 3)^5)^2} = \frac{x^2(-3x^3 - 1)^2(-33x^4 - 81 - 20x)}{(-x^4 + 3)^6}$$

$$28) f'(x) = 4((2x^5 - 5)^3 + 3)^3 \cdot 3(2x^5 - 5)^2 \cdot 10x^4 = 120x^4((2x^5 - 5)^3 + 3)^3 \cdot (2x^5 - 5)^2 \quad 29) f'(x) = \sec^2(\sin x^4) \cdot \cos x^4 \cdot 4x^3 = 4x^3 \cdot \sec^2(\sin x^4) \cos x^4$$

$$30) f'(x) = -\csc 3x^2 \cot 3x^2 \cdot 6x = -6x \csc 3x^2 \cdot \cot 3x^2 \quad 31) f'(x) = -\csc^2 x^4 \cdot 4x^3 = -4x^3 \cdot \csc^2 x^4$$

$$32) f'(x) = -\csc^2 2x^2 \cdot 4x = -4x \cdot \csc^2 2x^2 \quad 33) f^{(4)}(x) = -480x \quad 34) f'''(x) = 96x$$

$$35) f'''(x) = -300x^2 + 24x \quad 36) f'''(x) = 0$$

$$37) -\frac{8}{25}$$

$$38) -1$$

$$39) \frac{\sqrt{6}}{4}$$

$$40) 2$$