

Differentiation Review #2 (Rates of Change, Higher Order Derivatives, Implicit)

For each problem, find the average rate of change of the function over the given interval.

1) $y = -\frac{1}{x+3}; [-1, 2]$

2) $y = x^2 - 2x - 1; [-2, -1]$

3) $y = -2x^2 + 2x - 1; [-1, 2]$

4) $y = -\frac{1}{x-3}; [-1, 0]$

For each problem, find the instantaneous rate of change of the function at the given value.

5) $y = -x^2 + 1; -1$

6) $y = -x^2 - 1; -1$

7) $y = x^2 - 2; -2$

8) $y = \frac{1}{x-2}; -2$

For each problem, find the indicated derivative with respect to x .

9) $f(x) = 3x^3$ Find f'''

10) $f(x) = -2x^{\frac{5}{2}}$ Find f'''

11) $f(x) = 2x^{\frac{5}{3}} + 3x^{\frac{4}{5}} - \frac{1}{x^2}$ Find $f^{(4)}$

12) $f(x) = -x^{-2} + 5x^{-3}$ Find f''

13) $f(x) = -3x^{-2} + 3x^{-3} - \frac{3}{x^4}$ Find $f^{(4)}$

14) $f(x) = -\frac{1}{x^2}$ Find f'''

Differentiate each function with respect to x .

15) $f(x) = \cot 3x^2$

16) $f(x) = \csc x^4$

17) $f(x) = \cos 5x^5$

18) $f(x) = \sin 5x^4$

19) $f(x) = \csc 4x^3$

20) $f(x) = \csc x^3$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

21) $5 = 3x - 2y^2$

22) $3 = 4x^3 + 3x^2y^3$

23) $4x^2y^2 + 2 = 4x$

24) $2 = 4x^2 - 4y^3$

25) $-5xy + 5y^3 = x^2$

26) $-5x^2y^2 + 5y = 2x^3$

27) $x^2 + 5y = 4x^2y$

28) $1 = 4x^3 - xy$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y .

29) $3 = 2x^3 - 4y^2$

30) $x^3 = 4y^2 + 2$

31) $2 = x^2 - 2y^2$

32) $5 = x - y^2$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ at the given point.

33) $4x + 3y^2 = 2y^3$ at $(1, 2)$

34) $x = -xy + 4$ at $(-1, -5)$

35) $4y = 3x - 2y^3$ at $(-2, -1)$

36) $x^3 - 2xy^3 = 4$ at $(2, 1)$

Answers to Differentiation Review #2 (Rates of Change, Higher Order Derivatives, Implicit)

$$1) \frac{1}{10}$$

$$2) -5$$

$$3) 0$$

$$4) \frac{1}{12}$$

$$5) 2$$

$$6) 2$$

$$7) -4$$

$$8) -\frac{1}{16}$$

$$9) f'''(x) = 18$$

$$10) f'''(x) = -\frac{15}{4x^2}$$

$$11) f^{(4)}(x) = \frac{80}{81x^3} - \frac{792}{625x^5} - \frac{120}{x^6}$$

$$12) f'''(x) = -\frac{6}{x^4} + \frac{60}{x^5}$$

$$13) f^{(4)}(x) = -\frac{360}{x^6} + \frac{1080}{x^7} - \frac{2520}{x^8}$$

$$14) f'''(x) = \frac{24}{x^5}$$

$$15) f'(x) = -\csc^2 3x^2 \cdot 6x$$

$$= -6x \cdot \csc^2 3x^2$$

$$16) f'(x) = -\csc x^4 \cot x^4 \cdot 4x^3$$

$$= -4x^3 \csc x^4 \cdot \cot x^4$$

$$17) f'(x) = -\sin 5x^5 \cdot 25x^4$$

$$= -25x^4 \sin 5x^5$$

$$18) f'(x) = \cos 5x^4 \cdot 20x^3$$

$$= 20x^3 \cos 5x^4$$

$$19) f'(x) = -\csc 4x^3 \cot 4x^3 \cdot 12x^2$$

$$= -12x^2 \csc 4x^3 \cdot \cot 4x^3$$

$$20) f'(x) = -\csc x^3 \cot x^3 \cdot 3x^2$$

$$= -3x^2 \csc x^3 \cdot \cot x^3$$

$$21) \frac{dy}{dx} = \frac{3}{4y}$$

$$22) \frac{dy}{dx} = \frac{-4x - 2y^3}{3xy^2}$$

$$23) \frac{dy}{dx} = \frac{1 - 2xy^2}{2x^2y}$$

$$24) \frac{dy}{dx} = \frac{2x}{3y^2}$$

$$25) \frac{dy}{dx} = \frac{2x + 5y}{-5x + 15y^2}$$

$$26) \frac{dy}{dx} = \frac{6x^2 + 10xy^2}{-10x^2y + 5}$$

$$27) \frac{dy}{dx} = \frac{8xy - 2x}{5 - 4x^2}$$

$$28) \frac{dy}{dx} = \frac{12x^2 - y}{x}$$

$$29) \frac{d^2y}{dx^2} = \frac{24xy^2 - 9x^4}{16y^3}$$

$$30) \frac{d^2y}{dx^2} = \frac{48xy^2 - 9x^4}{64y^3}$$

$$31) \frac{d^2y}{dx^2} = \frac{2y^2 - x^2}{4y^3}$$

$$32) \frac{d^2y}{dx^2} = -\frac{1}{4y^3}$$

$$33) \left. \frac{dy}{dx} \right|_{\substack{x=1 \\ y=2}} = \frac{1}{3}$$

$$34) \left. \frac{dy}{dx} \right|_{\substack{x=-1 \\ y=-5}} = -4$$

$$35) \left. \frac{dy}{dx} \right|_{\substack{x=-2 \\ y=-1}} = \frac{3}{10}$$

$$36) \left. \frac{dy}{dx} \right|_{\substack{x=2 \\ y=1}} = \frac{5}{6}$$