

$$\int \frac{1}{3x} dx = \frac{1}{3} \int \frac{1}{x} dx = \frac{1}{3} \ln|x| + C$$

$$\int \sqrt{6x} dx = \int \frac{\sqrt{u}}{6} du = \frac{1}{6} \int u^{1/2} du$$

$$u = 6x$$

$$du = 6dx$$

$$\frac{du}{6} = dx$$

$$= \frac{1}{6} \left[\frac{u^{3/2}}{3/2} \right] = \frac{1}{6} \left[\frac{2u^{3/2}}{3} \right]$$

$$= \frac{u^{3/2}}{9} = \frac{(6x)^{3/2}}{9} + C$$

$$\int_2^5 \frac{5x^4}{x^5-4} dx = \int_2^5 \frac{du}{u} = \ln|u|_2^5$$

$$u = x^5 - 4$$

$$du = 5x^4 dx$$

$$= \ln|x^5-4|_2^5$$

$$= \ln|5^5-4| - \ln|2^5-4|$$

$$= \ln|3125-4| - \ln|32-4|$$

$$= \ln|3121| - \ln|28| = \ln \left| \frac{3121}{28} \right|$$

Implicit Differentiation

$$\begin{aligned}x + y &= 4 \\ dx + dy &= 0 \\ \frac{-dx}{-dx} & \quad \frac{-dx}{-dx} \\ \frac{dy}{dx} &= \frac{dx}{dx} \\ \frac{dy}{dx} &= -1\end{aligned}$$

$$\begin{aligned}4x + 5y &= 10 \\ 4dx + 5dy &= 0 \\ \frac{-4dx}{-4dx} & \quad \frac{-4dx}{-4dx} \\ \frac{5dy}{dx} &= \frac{-4dx}{dx} \\ \frac{5dy}{5} &= \frac{-4}{5} \\ \frac{dy}{dx} &= \frac{-4}{5}\end{aligned}$$

$$\begin{aligned}xy &= 6 \\ ydx + xdy &= 0 \\ \frac{-ydx}{-ydx} & \quad \frac{-ydx}{-ydx} \\ xdy &= -ydx \\ \frac{dy}{dx} &= \frac{-y}{x}\end{aligned}$$

$$\begin{aligned}5xy &= 3 \\ 5ydx + 5xdy &= 0 \\ 5xdy &= -5ydx \\ \frac{dy}{dx} &= \frac{-5y}{5x} = \frac{-y}{x}\end{aligned}$$

$$\begin{aligned}x^2 + y^2 &= 16 \\ 2xdx + 2ydy &= 0 \\ \frac{dy}{dx} &= \frac{-y}{x}\end{aligned}$$

$$\begin{aligned}3x^2 + 2xy &= 4 \\ 6xdx + 2ydx + 2xdy &= 0 \\ (6x + 2y)dx + 2xdy &= 0 \\ \frac{2xdy}{2xdx} &= \frac{-(6x + 2y)dx}{2xdx} \\ \frac{dy}{dx} &= \frac{-(6x + 2y)}{2x}\end{aligned}$$

$$\begin{aligned}2y^2 &= 3x^2y^2 + 5x \\ 4ydy &= 6xy^2dx + \boxed{6x^2ydy} + 5dx \\ 4ydy - 6x^2ydy &= 6xy^2dx + 5dx \\ (4y - 6x^2y)dy &= (6xy^2 + 5)dx \\ \frac{dy}{dx} &= \frac{6xy^2 + 5}{4y - 6x^2y}\end{aligned}$$