Calculus

Integration

Evaluate each indefinite integral.

1) $\int \left(\frac{6}{x^4} + \frac{4}{x^5}\right) dx$

$\int 6x^{-4} + 4x^{-5} dx$

$\frac{6x^{-3}}{-3} + \frac{4x^{-4}}{-4} = -\frac{2}{x^3} - \frac{1}{x^4} + C$

2) $\int \left(3x^2 - \frac{20}{x^5}\right) dx$

3) $\int \left(-2x^{-2} - 20x^{-5}\right) dx$

4) $\int \left(6x^5 + 15x^2 - \frac{8}{x^5}\right) dx$

5) $\int \left(4x + \frac{8}{x^3}\right) dx$

6) $\int \left(30x^5 + 15x^2\right) dx$

7) $\int -2 \cdot \csc^2 x \, dx$

8) $\int \sec^2 x \, dx$

9) $\int -4 \cdot \sec^2 x \, dx$

10) $\int -\frac{5}{\cos^2 x} dx$

11) $\int -\frac{1}{\sec x} dx$

12) $\int 3\sec x \cdot \tan x \, dx$

13) $\int 15x^2(5x^3 - 1)^5\, dx$

$u = 5x^3 - 1$

$du = 15x^2\, dx$

$\int u^5\, du = \frac{u^6}{6} = \frac{(5x^3-1)^6}{6} + C$

14) $\int 2\sin -2x \cdot \cos^4 -2x\, dx$

$u = \cos(-2x)$

$du = 2\sin(-2x)\, dx$

$\int u^4\, du = \frac{u^5}{5} = \frac{\cos^5(-2x)}{5} + C$

15) $\int 20x^3(5x^4 - 3)^5\, dx$

$u = 5x^4 - 3$

$du = 20x^3\, dx$

$\int u^5\, du = \frac{u^6}{6} = \frac{(5x^4-3)^6}{6} + C$

16) $\int (\tan 5x)^{-3} \cdot 5 \cdot \sec^2 5x\, dx$

$u = \tan 5x$

$\boxed{du} = 5\sec^2 5x\, dx$

$\int u^{-3}\, du = \frac{u^{-2}}{-2} = \frac{(\tan 5x)^{-2}}{-2} = -\frac{1}{2(\tan 5x)^2} + C$

17) $\int -\sin x \cdot \cos^3 x\, dx$

18) $\int 3 \cdot \csc^2 -3x \cdot \cot^5 -3x\, dx$

Evaluate each definite integral.

19) $\int_0^3 (-x^3 + 3x^2 - 1)\, dx$

20) $\int_0^3 (x^3 - 2x^2 - 2)\, dx$

21) $\int_0^{\frac{\pi}{2}} -2\cos x\, dx$

22) $\int_{-2}^{3} (-2x - 1)\, dx$

$u = \boxed{5x^4 + 3}$

$du = \boxed{\underbrace{\cancel{20}}_{19}x^3}\,\boxed{dx}$

$\dfrac{du}{19x^3} = dx \qquad \dfrac{20x^{\cancel{3}}}{19\cancel{x^3}}$

$u^5\,du$

$\int \boxed{\cancel{20x^3}}\,(\boxed{5x^4+3})^5\,\boxed{\cancel{dx}}$

$20x^3\,dx = du$

$$5x^2y^2 + 4xy^3 = 2x^3$$

$$10xy^2\,\boxed{dx} + 10x^2y\,dy + 4y^3\,\boxed{dx} + 12xy^2\,dy = 6x^2\,\boxed{dx}$$

$$-10xy^2dx \qquad\qquad -4y^3dx$$

$$10x^2y\,\boxed{dy} + 12xy^2\,\boxed{dy} = 6x^2\,\boxed{dx} - 10xy^2\,\boxed{dx} - 4y^3\,\boxed{dx}$$

$$\frac{\cancel{(10x^2y+12xy^2)}\,dy}{\cancel{(10x^2y+12xy^2)}\,dx} = \frac{(6x^2-10xy^2-4y^3)\,\cancel{dx}}{(10x^2y+12xy^2)\,\cancel{dx}}$$

$$\frac{dy}{dx} = \frac{6x^2-10xy^2-4y^3}{10x^2y+12xy^2}$$

23) $\displaystyle\int_{-\frac{3\pi}{4}}^{-\frac{\pi}{3}} -2\sin x\,dx$

24) $\displaystyle\int_{-1}^{2} \left(-x^3 + x^2 + 1\right) dx$

For each problem, approximate the area under the curve over the given interval using 4 le endpoint rectangles.

25) $y = -x^2 + 13$; $[-2, 2]$

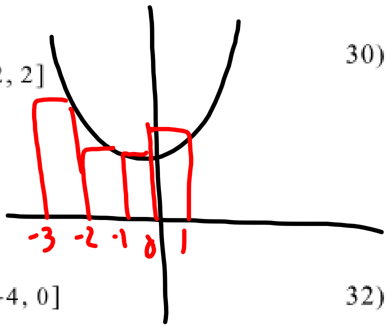
26) $y = \dfrac{x}{2} + 6$; $[-4, 4]$

27) $y = -x + 5$; $[-6, -2]$

28) $y = x^2 + 2$; $[-1, 3]$

For each problem, approximate the area under the curve over the given interval using 4 ri endpoint rectangles.

29) $y = -\dfrac{x^2}{2} - x + 5$; $[-2, 2]$



30) $y = x^2 + 2x + 2$; $[-3, 1]$

$\displaystyle\int_{-3}^{1} x^2+2x+2\,dx$

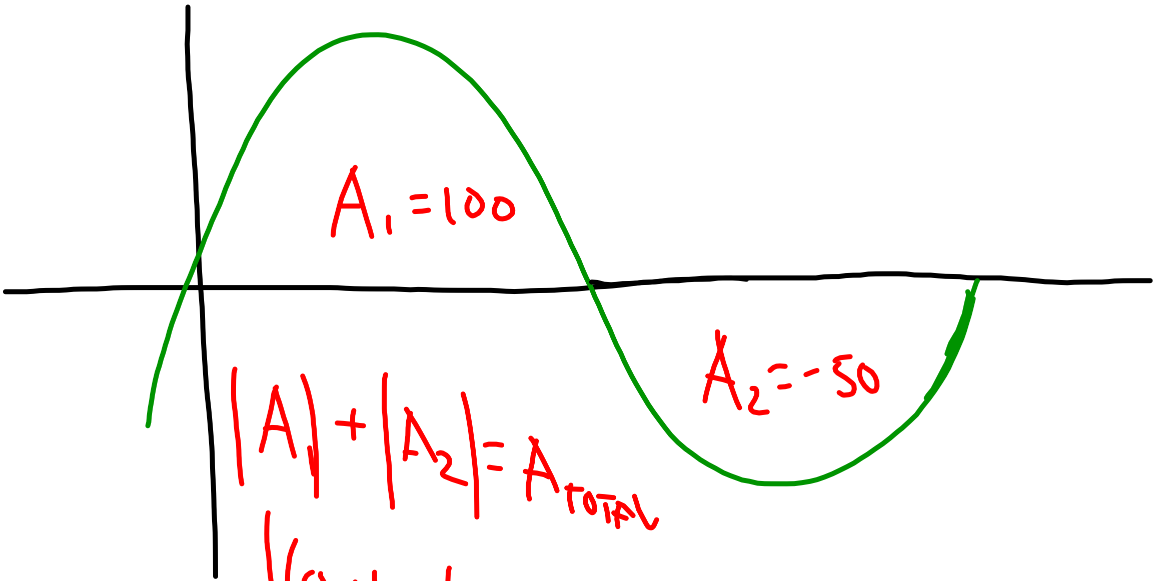
31) $y = -x^2 - 2x + 11$; $[-4, 0]$

32) $y = -x^2 + 13$; $[-2, 2]$

For each problem, find the area under the curve over the given interval.

33) $y = \sin x$; $[\frac{\pi}{4}, \frac{2\pi}{3}]$

34) $y = \cos x$; $[0, \frac{\pi}{2}]$



$$|A_1| + |A_2| = A_{TOTAL}$$

$$|100| + |-50| = 100 + 50 = 150$$

35) $y = 2\sqrt{x}$; $[5, 7]$

36) $y = \frac{4}{x^2}$; $[-3, -1]$

Answers to Integration

1) $-\frac{2}{x^3} - \frac{1}{x^4} + C$
2) $x^3 + \frac{5}{x^4} + C$
3) $\frac{2}{x} + \frac{5}{x^4} + C$
4) $x^6 + 5$
5) $2x^2 - \frac{4}{x^2} + C$
6) $5x^6 + 5x^3 + C$
7) $2\cot x + C$
8) $\tan x$
9) $-4\tan x + C$
10) $-5\tan x + C$
11) $-\sin x + C$
12) $3\sec$
13) $\frac{1}{6}(5x^3 - 1)^6 + C$
14) $\frac{1}{5} \cdot \cos^5 -2x + C$
15) $\frac{1}{6}(5x^4 - 3)^6 + C$
16) $-\frac{}{2}$
17) $\frac{1}{4} \cdot \cos^4 x + C$
18) $\frac{1}{6} \cdot \cot^6 -3x + C$
19) $\frac{15}{4} = 3.75$
20) $-\frac{15}{4}$
21) -2
22) -10
23) $1 + \sqrt{2} \approx 2.414$
24) $\frac{9}{4} =$
25) 46
26) 44
27) 38
28) 14
29) 15
30) 10
31) 42
32) 46
33) $\frac{1 + \sqrt{2}}{2} \approx 1.207$
34) 1
35) $\frac{4(7\sqrt{7} - 5\sqrt{5})}{3} \approx 9.787$
36) $\frac{8}{3} \approx 2.667$

