Calculus

Integration Review #2

Evaluate each indefinite integral.

1) $\int -9x^2 \, dx$

2) $\int 12x^2 \, dx$

3) $\int \left(3x^2 + 2\right) dx$

4) $\int 8x\left(2x^2 - 1\right) dx$

5) $\int \left(-6x^2 + 8x - 3\right) dx$

6) $\int \left(30x^5 + 5x^4 + 3x^2\right) dx$

7) $\int 2\cos x \, dx$

8) $\int 4\csc x \cdot \cot x \, dx$

9) $\int -4\cos x \, dx$

10) $\int 2 \cdot \csc^2 x \, dx$

11) $\int \left(3x^5 - 2\right)^4 \cdot 15x^4 \, dx$

12) $\int \left(x^3 + 2\right)^4 \cdot 3x^2 \, dx$

13) $\int \left(4x^2 + 5\right)^4 \cdot 8x \, dx$

14) $\int 10x\left(5x^2 + 2\right)^4 \, dx$

15) $\int (4x - 1)^4 \cdot x \, dx$

16) $\int 3x(5x - 1)^4 \, dx$

17) $\int -24x^2 \sin (2x^3 - 3) \, dx$

18) $\int 30x^4 \cdot \sec^2 (2x^5 + 1) \, dx$

19) $\int -12x^2 \cdot \csc^2 (x^3 - 4) \, dx$

20) $\int -24x \cdot \sec^2 (3x^2 + 2) \, dx$

For each problem, approximate the area under the curve over the given interval using 4 left endpoint rectangles. For comparison, also find the exact area using integration.

21) $y = -x^2 + 12; \ [-2, 2]$

22) $y = x + 6; \ [-5, 3]$

23) $y = -\dfrac{x}{2} + 5; \ [-7, -3]$

24) $y = -x + 6; \ [-4, 0]$

For each problem, approximate the area under the curve over the given interval using 4 right endpoint rectangles. For comparison, also find the exact area using integration.

25) $y = -\dfrac{x^2}{2} - x + 5; \ [-3, 1]$

26) $y = x^2 + 2x + 2; \ [-4, 0]$

27) $y = -x + 6; \ [0, 4]$

28) $y = -\dfrac{x}{2} + 5; \ [3, 7]$

Evaluate each definite integral.

29) $\displaystyle\int_{0}^{3} \left(-x^3 + 4x^2 - 7\right)\, dx$

30) $\displaystyle\int_{-1}^{3} \left(-x^3 + 2x^2 + 2\right)\, dx$

31) $\displaystyle\int_{2}^{6} x\, dx$

32) $\displaystyle\int_{-7}^{-2} -x\, dx$

33) $\displaystyle\int_{2}^{3} \frac{1}{x^3}\, dx$

34) $\displaystyle\int_{2}^{3} 4x^{\frac{1}{3}}\, dx$

35) $\displaystyle\int_{0}^{4} 4(x+1)^{\frac{1}{3}}\, dx$

36) $\displaystyle\int_{1}^{4} 4x^{\frac{1}{3}}\, dx$

37) $\displaystyle\int_{-2}^{1} \frac{4x}{\left(x^2 + 2\right)^2}\, dx$

38) $\displaystyle\int_{-1}^{0} -\frac{16x}{\left(4x^2 + 1\right)^2}\, dx$

39) $\displaystyle\int_{-1}^{0} \frac{4x}{\left(2x^2 + 2\right)^2}\, dx$

40) $\displaystyle\int_{-2}^{0} \frac{16x}{\left(4x^2 + 4\right)^2}\, dx$

41) $\displaystyle\int_{0}^{1} -18x^2\left(3x^3 - 1\right)^2\, dx$

42) $\displaystyle\int_{1}^{2} -\frac{4x}{\left(x^2 + 1\right)^2}\, dx$

Answers to Integration Review #2

1) $-3x^3 + C$

2) $4x^3 + C$

3) $x^3 + 2x + C$

4) $4x^4 - 4x^2 + C$

5) $-2x^3 + 4x^2 - 3x + C$

6) $5x^6 + x^5 + x^3 + C$

7) $2\sin x + C$

8) $-4\csc x + C$

9) $-4\sin x + C$

10) $-2\cot x + C$

11) $\frac{1}{5}(3x^5 - 2)^5 + C$

12) $\frac{1}{5}(x^3 + 2)^5 + C$

13) $\frac{1}{5}(4x^2 + 5)^5 + C$

14) $\frac{1}{5}(5x^2 + 2)^5 + C$

15) $\frac{1}{96}(4x - 1)^6 + \frac{1}{80}(4x - 1)^5 + C$

16) $\frac{1}{50}(5x - 1)^6 + \frac{3}{125}(5x - 1)^5 + C$

17) $4\cos(2x^3 - 3) + C$

18) $3\tan(2x^5 + 1) + C$

19) $4\cot(x^3 - 4) + C$

20) $-4\tan(3x^2 + 2) + C$

21) Exact: $\frac{128}{3} \approx 42.667$

Approximate: 42

22) Exact: 40

Approximate: 32

23) Exact: 30

Approximate: 31

24) Exact: 32

Approximate: 34

25) Exact: $\frac{58}{3} \approx 19.333$

Approximate: 19

26) Exact: $\frac{40}{3} \approx 13.333$

Approximate: 10

27) Exact: 16

Approximate: 14

28) Exact: 10

Approximate: 9

29) $-\frac{21}{4} = -5.25$

30) $\frac{20}{3} \approx 6.667$

31) 16

32) $\frac{45}{2} = 22.5$

33) $\frac{5}{72} \approx 0.069$

34) $9\sqrt[3]{3} - 6\sqrt[3]{2} \approx 5.421$

35) $15\sqrt[3]{5} - 3 \approx 22.65$

36) $12\sqrt[3]{4} - 3 \approx 16.049$

37) $-\frac{1}{3} \approx -0.333$

38) $\frac{8}{5} = 1.6$

39) $-\frac{1}{4} = -0.25$

40) $-\frac{2}{5} = -0.4$

41) -6

42) $-\frac{3}{5} = -0.6$