Calculus

Last Test Before Final Review

Evaluate each limit.

1) $\lim\limits_{x \to -1} \left(-x^3 - x^2 + x + 4\right)$

2) $\lim\limits_{x \to -3} \left(-x^3 - 2x^2 + 4x + 3\right)$

3) $\lim\limits_{x \to -2} -\dfrac{x}{x+1}$

4) $\lim\limits_{x \to -1} \sqrt{-2x+5}$

5) $\lim\limits_{x \to -\frac{\pi}{6}} -2\cos\left(x\right)$

6) $\lim\limits_{x \to 1} \left(x^3 - x^2 + 2\right)$

7) $\lim\limits_{x \to -2^+} f(x),\ f(x) = \begin{cases} -\dfrac{x}{2} - 1, & x < -2 \\ 0, & x \geq -2 \end{cases}$

8) $\lim\limits_{x \to -3^-} f(x),\ f(x) = \begin{cases} -x^2 - 2x - 1, & x < -3 \\ -\dfrac{x}{2} - \dfrac{1}{2}, & x \geq -3 \end{cases}$

9) $\lim\limits_{x \to 0^+} f(x),\ f(x) = \begin{cases} 4, & x \leq 0 \\ x^2 + 4x + 4, & x > 0 \end{cases}$

10) $\lim\limits_{x \to 0} f(x),\ f(x) = \begin{cases} -x^2 - 2x - 2, & x < 0 \\ 2x - 2, & x \geq 0 \end{cases}$

11) $\lim\limits_{x \to 0} \dfrac{1 - \sin\left(\dfrac{\pi}{2} - x\right)}{x}$

12) $\lim\limits_{x \to 0} \dfrac{\sin x}{x}$

13) $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$

14) $\lim_{x \to 1} \frac{x - 1}{\sqrt{x + 8} - 3}$

15) $\lim_{x \to -2} \frac{x + 2}{x^2 + 5x + 6}$

16) $\lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1}$

17) $\lim_{x \to 0} -\cot(2x)$

18) $\lim_{x \to -\pi} -2\sec(2x)$

19) $\lim_{x \to -2} \frac{x - 1}{x^2 + 4x + 4}$

20) $\lim_{x \to -\frac{3\pi}{4}^-} 2\tan(2x)$

21) $\lim_{x \to 1^-} \frac{x - 1}{x^2 - 4x + 3}$

22) $\lim_{x \to -\frac{\pi}{2}} \tan(x)$

23) $\lim_{x \to -\infty} -\frac{9x}{x^2 + 9}$

24) $\lim_{x \to -\infty} \sqrt{\frac{3x^2 + 4}{2x^2 + 3}}$

25) $\lim_{x \to \infty} \frac{-x - 1}{x^2 + 2x + 2}$

26) $\lim_{x \to -\infty} -\frac{x^3}{x^2 - 2}$

27) $\lim_{x \to \infty} \frac{16x}{x^2 + 16}$

28) $\lim_{x \to \infty} -\frac{25}{x^2 + 5}$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of *x* and *y*.

29) $5x^2y^2 + 4xy^3 = 2x^3$

30) $x^3 - 3x^2y = 4$

31) $5y + y^2 = 2x^3$

32) $x^3 = 4x^3y^3 + y^3$

33) $5x^2y^2 = 5x^2 - 5x^3y^2$

34) $-4xy^3 + 1 = 5x^2$

35) $5x - 2y^2 = y$

36) $5x = -4y + y^3$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ at the given point.

37) $2x^2y^3 = 4x^2 - 2y$ at $(-1, 1)$

38) $-3x^2y + 5 = 2x^3$ at $(1, 1)$

39) $3 = 4x^2 + x^2y^3$ at $(1, -1)$

40) $3x^3 = -2x^2y + 2xy$ at $(2, -6)$

Solve each optimization problem.

41) An architect is designing a composite window by attaching a semicircular window on top of a rectangular window, so the diameter of the top window is equal to and aligned with the width of the bottom window. If the architect wants the perimeter of the composite window to be 18 ft, what dimensions should the bottom window be in order to create the composite window with the largest area?

42) A geometry student wants to draw a rectangle inscribed in the ellipse $x^2 + 4y^2 = 16$. What is the area of the largest rectangle that the student can draw?

43) A supermarket employee wants to construct an open-top box from a 10 by 16 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

44) Which points on the graph of $y = 5 - x^2$ are closest to the point $(0, 3)$?

45) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 1372 ft³ of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

For each problem, find the area under the curve over the given interval.

46) $y = \frac{x^2}{2} - 4x + 9;\ [0, 5]$

47) $y = \sqrt{x};\ [5, 6]$

48) $y = x^2 - 4x + 6;\ [1, 4]$

49) $y = \frac{3}{x^2};\ [2, 3]$

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the the *x*-axis.

50) $y = x^2 + 2,\ y = 0,\ x = 1,\ x = 2$

51) $y = \sqrt{x},\ y = 0,\ x = 4$

52) $y = -x^2 + 4,\ y = 0,\ x = -1,\ x = 2$

53) $y = \sqrt{x + 3},\ y = 0,\ x = 1,\ x = 3$

Answers to Last Test Before Final Review

1) 3
2) 0
3) -2
4) $\sqrt{7}$
5) $-\sqrt{3}$
6) 2
7) 0
8) -4
9) 4
10) -2
11) 0
12) 1
13) $\frac{1}{6}$
14) 6
15) 1
16) $\frac{1}{2}$
17) Does not exist.
18) -2
19) $-\infty$
20) ∞
21) $-\frac{1}{2}$
22) Does not exist.
23) 0
24) $\frac{\sqrt{6}}{2}$
25) 0
26) ∞
27) 0
28) 0

29) $\frac{dy}{dx} = \frac{3x^2 - 5y^2x - 2y^3}{5x^2y + 6xy^2}$
30) $\frac{dy}{dx} = \frac{x - 2y}{x}$
31) $\frac{dy}{dx} = \frac{6x^2}{5 + 2y}$
32) $\frac{dy}{dx} = \frac{-4x^2y^3 + x^2}{4y^2x^3 + y^2}$
33) $\frac{dy}{dx} = \frac{2 - 3xy^2 - 2y^2}{2xy + 2x^2y}$
34) $\frac{dy}{dx} = \frac{-5x - 2y^3}{6xy^2}$
35) $\frac{dy}{dx} = -\frac{5}{-4y - 1}$
36) $\frac{dy}{dx} = \frac{5}{-4 + 3y^2}$
37) $\left.\frac{dy}{dx}\right|_{\substack{x=-1\\y=1}} = -\frac{1}{2}$
38) $\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=1}} = -4$
39) $\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=-1}} = -2$
40) $\left.\frac{dy}{dx}\right|_{\substack{x=2\\y=-6}} = 0$

41) A = the area of the composite window x = the width of the bottom window = the diameter of the top window
Function to maximize: $A = x\left(\frac{18}{2} - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{1}{2}\pi \cdot \left(\frac{x}{2}\right)^2$ where $0 < x < \frac{72}{4 + \pi}$
Dimensions of the bottom window: $\frac{36}{4 + \pi}$ ft (width) by $\frac{18}{4 + \pi}$ ft (height)

42) A = the area of the rectangle x = half the base of the rectangle
Function to maximize: $A = 2x \cdot 2 \cdot \frac{\sqrt{16 - x^2}}{2}$ where $0 < x < 4$
Area of largest rectangle: 16

43) V = the volume of the box x = the length of the sides of the squares
Function to maximize: $V = (16 - 2x)(10 - 2x) \cdot x$ where $0 < x < 5$
Sides of the squares: 2 in

44) d = the distance from point $(0, 3)$ to a point on the parabola x = the x-coord. of a point on the parabola
Function to minimize: $d = \sqrt{x^2 + (5 - x^2 - 3)^2}$ where $-\infty < x < \infty$
Points on the parabola that are closest to the point $(0, 3)$: $\left(-\frac{\sqrt{6}}{2}, \frac{7}{2}\right), \left(\frac{\sqrt{6}}{2}, \frac{7}{2}\right)$

45) A = the area of the glass x = the length of the sides of the square bottom
Function to minimize: $A = x^2 + 4x \cdot \frac{1372}{x^2}$ where $0 < x < \infty$
Dimensions of the aquarium: 14 ft by 14 ft by 7 ft tall

46) $\int_0^5 \left(\frac{x^2}{2} - 4x + 9\right) dx$
$= \frac{95}{6} \approx 15.833$

47) $\int_5^6 \sqrt{x}\, dx$
$= \frac{2\left(6\sqrt{6} - 5\sqrt{5}\right)}{3} \approx 2.344$

48) $\int_1^4 (x^2 - 4x + 6)\, dx$
$= 9$

49) $\int_{2}^{3} \frac{3}{x^2}\,dx$

$= \frac{1}{2} = 0.5$

50) $\pi\int_{1}^{2} \left(x^2 + 2\right)^2\,dx$

$= \frac{293}{15}\pi \approx 61.366$

51) $\pi\int_{0}^{4} \left(\sqrt{x}\right)^2\,dx$

$= 8\pi \approx 25.133$

52) $\pi\int_{-1}^{2} \left(-x^2 + 4\right)^2\,dx$

$= \frac{153}{5}\pi \approx 96.133$

53) $\pi\int_{1}^{3} \left(\sqrt{x + 3}\right)^2\,dx$

$= 10\pi \approx 31.416$

