Calculus

Lower Sums Extra Practice

For each problem, find the area under the curve over the given interval. Set up your solution using the limit as *n* goes to ∞ of the lower sum.

1) $f(x) = x^2$; $[0, 1]$

2) $f(x) = 3x$; $[0, 1]$

3) $f(x) = x^3 + 5$; $[0, 2]$

4) $f(x) = x^3 + 2$; $[0, 1]$

5) $f(x) = x$; $[0, 9]$

6) $f(x) = x^3$; $[0, 1]$

Answers to Lower Sums Extra Practice

1) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \left(0 + \frac{1-0}{n}(k-1)\right)^2 \cdot \frac{1-0}{n}$

$\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \left(\frac{k^2}{n^3} - \frac{2k}{n^3} + \frac{1}{n^3}\right)$

$\lim\limits_{n\to\infty} \left(\frac{1}{n^3} \cdot \sum\limits_{k=1}^{n} k^2 - \frac{2}{n^3} \cdot \sum\limits_{k=1}^{n} k + \frac{1}{n^3} \cdot \sum\limits_{k=1}^{n} 1\right)$

$\lim\limits_{n\to\infty} \left(\frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{2}{n^3} \cdot \frac{n(n+1)}{2} + \frac{1}{n^3} \cdot n\right)$

$\lim\limits_{n\to\infty} \left(\frac{1}{3} - \frac{1}{2n} + \frac{1}{6n^2}\right)$

$\frac{1}{3} \approx 0.333$

2) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} 3\left(0 + \frac{1-0}{n}(k-1)\right) \cdot \frac{1-0}{n}$

$\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \left(\frac{3k}{n^2} - \frac{3}{n^2}\right)$

$\lim\limits_{n\to\infty} \left(\frac{3}{n^2} \cdot \sum\limits_{k=1}^{n} k - \frac{3}{n^2} \cdot \sum\limits_{k=1}^{n} 1\right)$

$\lim\limits_{n\to\infty} \left(\frac{3}{n^2} \cdot \frac{n(n+1)}{2} - \frac{3}{n^2} \cdot n\right)$

$\lim\limits_{n\to\infty} \left(\frac{3}{2} - \frac{3}{2n}\right)$

$\frac{3}{2} = 1.5$

3) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \left(\left(0 + \frac{2-0}{n}(k-1)\right)^3 + 5\right) \cdot \frac{2-0}{n}$

$\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \left(\frac{10}{n} + \frac{16k^3}{n^4} - \frac{48k^2}{n^4} + \frac{48k}{n^4} - \frac{16}{n^4}\right)$

$\lim\limits_{n\to\infty} \left(\frac{10}{n} \cdot \sum\limits_{k=1}^{n} 1 + \frac{16}{n^4} \cdot \sum\limits_{k=1}^{n} k^3 - \frac{48}{n^4} \cdot \sum\limits_{k=1}^{n} k^2 + \frac{48}{n^4} \cdot \sum\limits_{k=1}^{n} k - \frac{16}{n^4} \cdot \sum\limits_{k=1}^{n} 1\right)$

$\lim\limits_{n\to\infty} \left(\frac{10}{n} \cdot n + \frac{16}{n^4} \cdot \frac{n^2 \cdot (n+1)^2}{4} - \frac{48}{n^4} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{48}{n^4} \cdot \frac{n(n+1)}{2} - \frac{16}{n^4} \cdot n\right)$

$\lim\limits_{n\to\infty} \left(14 - \frac{8}{n} + \frac{4}{n^2}\right)$

14

4) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\left(0 + \frac{1-0}{n}(k-1)\right)^{3} + 2\right) \cdot \frac{1-0}{n}$

$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{2}{n} + \frac{k^3}{n^4} - \frac{3k^2}{n^4} + \frac{3k}{n^4} - \frac{1}{n^4}\right)$

$\lim_{n\to\infty} \left(\frac{2}{n} \cdot \sum_{k=1}^{n} 1 + \frac{1}{n^4} \cdot \sum_{k=1}^{n} k^3 - \frac{3}{n^4} \cdot \sum_{k=1}^{n} k^2 + \frac{3}{n^4} \cdot \sum_{k=1}^{n} k - \frac{1}{n^4} \cdot \sum_{k=1}^{n} 1\right)$

$\lim_{n\to\infty} \left(\frac{2}{n} \cdot n + \frac{1}{n^4} \cdot \frac{n^2 \cdot (n+1)^2}{4} - \frac{3}{n^4} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{3}{n^4} \cdot \frac{n(n+1)}{2} - \frac{1}{n^4} \cdot n\right)$

$\lim_{n\to\infty} \left(\frac{9}{4} - \frac{1}{2n} + \frac{1}{4n^2}\right)$

$\frac{9}{4} = 2.25$

5) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(0 + \frac{9-0}{n}(k-1)\right) \cdot \frac{9-0}{n}$

$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{81k}{n^2} - \frac{81}{n^2}\right)$

$\lim_{n\to\infty} \left(\frac{81}{n^2} \cdot \sum_{k=1}^{n} k - \frac{81}{n^2} \cdot \sum_{k=1}^{n} 1\right)$

$\lim_{n\to\infty} \left(\frac{81}{n^2} \cdot \frac{n(n+1)}{2} - \frac{81}{n^2} \cdot n\right)$

$\lim_{n\to\infty} \left(\frac{81}{2} - \frac{81}{2n}\right)$

$\frac{81}{2} = 40.5$

6) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(0 + \frac{1-0}{n}(k-1)\right)^{3} \cdot \frac{1-0}{n}$

$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{k^3}{n^4} - \frac{3k^2}{n^4} + \frac{3k}{n^4} - \frac{1}{n^4}\right)$

$\lim_{n\to\infty} \left(\frac{1}{n^4} \cdot \sum_{k=1}^{n} k^3 - \frac{3}{n^4} \cdot \sum_{k=1}^{n} k^2 + \frac{3}{n^4} \cdot \sum_{k=1}^{n} k - \frac{1}{n^4} \cdot \sum_{k=1}^{n} 1\right)$

$\lim_{n\to\infty} \left(\frac{1}{n^4} \cdot \frac{n^2 \cdot (n+1)^2}{4} - \frac{3}{n^4} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{3}{n^4} \cdot \frac{n(n+1)}{2} - \frac{1}{n^4} \cdot n\right)$

$\lim_{n\to\infty} \left(\frac{1}{4} - \frac{1}{2n} + \frac{1}{4n^2}\right)$

$\frac{1}{4} = 0.25$