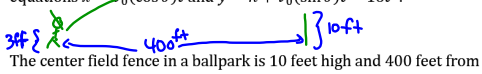
Parametric Equation Applications

Consider a projectile launched at a height h feet above the ground at an angle of θ with the horizontal. The initial velocity is v_0 feet per second and the path of the projectile is modeled by the parametric equations $x = v_0(\cos\theta)t$ and $y = h + v_0(\sin\theta)t - 16t^2$.



The center field fence in a ballpark is 10 feet high and 400 feet from home plate. A baseball is hit 3 feet above the ground. It leaves the bat at an angle of θ degrees with the horizontal at a speed of 100 miles per hour.

1. Write a set of parametric equations for the path of the baseball.

$\frac{100\text{ mi}}{\text{hr}} \cdot \frac{5280\text{ ft}}{1\text{ mi}}$ $x = 146.7t\cos\theta$ $y = 3 + 146.7t\sin\theta - 16t^2$

2. Use a graphing calculator to graph the path of the baseball for $\theta = 15°$. Is the hit a homerun?

$\frac{5280\text{ ft}}{\text{hr}} \cdot \frac{1\text{ hr}}{3600\text{ sec}}$ $V_0 = 146.7\ ft/sec$

$x = 146.7t\cos 15°$

$y = 3 + 146.7t\sin 15° - 16t^2$

3. Use a graphing calculator to graph the path of the baseball for $\theta = 23°$. Is the hit a homerun?

4. Find the minimum angle required for the hit to be a homerun.

$400 = x = 146.7t\cos\theta$

$10 = y = 3 + 146.7t\sin\theta - 16t^2$

$\frac{400}{146.7\cos\theta} = \frac{146.7t\cos\theta}{146.7\cos\theta} = t$

$10 = 3 + 146.7\left(\frac{400}{146.7\cos\theta}\right)\sin\theta - 16\left(\frac{400}{146.7\cos\theta}\right)^2$

$-10 \quad -10$

$0 = -7 + 400\tan\theta - 16\left(\frac{160000}{21520.9\cos^2\theta}\right)$

$\frac{1}{\cos^2\theta} \rightarrow \sec^2\theta \rightarrow \tan^2\theta + 1$

$0 = -7 + 400\tan\theta - \frac{2560000}{21520.9}(\tan^2\theta + 1)$

$0 = -7 + 400\tan\theta - 118.95\tan^2\theta - 118.95$

$0 = -125.95 + 400\tan\theta - 118.95\tan^2\theta$

$-118.95x^2 + 400x - 125.95 = 0$

$x = \tan\theta$

$\theta = \tan^{-1}\left(\frac{-400 \pm \sqrt{400^2 - 4(-118.95)(-125.95)}}{2(-118.95)}\right)$

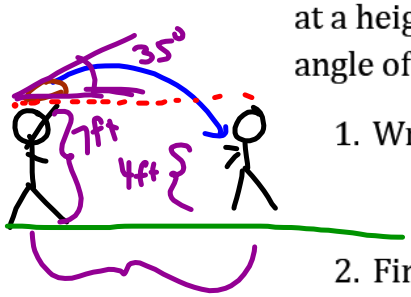
$\theta = \tan^{-1}\left(\frac{-400 \pm 316.343}{-237.9}\right)$

$\tan^{-1}\left(\frac{-83.657}{-237.9}\right)$ or $\tan^{-1}\left(\frac{-716.343}{-237.9}\right)$

$\tan^{-1}(.35)$ or $\tan^{-1}(3.01)$

$19.3°$ $71.6°$

The quarterback of a football team releases a pass at a height of 7 feet above the playing field, and the football is caught by a receiver at a height of 4 feet, 30 yards downfield. The pass is released at an angle of 35° with the horizontal.



1. Write a set of parametric equations for the path of the football.

$x = vt\cos 35$ $\quad y = 7 + vt\sin 35 - 16t^2$

2. Find the speed of the football when it is released.

$\frac{90}{v\cos 35} = \frac{vt\cos 35}{v\cos 35}$ $\quad 4 = 7 + vt\sin 35 - 16t^2$

$t = \frac{90}{v\cos 35}$

$4 = 7 + v\left(\frac{90}{v\cos 35}\right)\sin 35 - 16\left(\frac{90}{v\cos 35}\right)^2$

$0 = 3 + 90\tan 35 - \frac{16(8100)}{v^2\cos^2 35}$

$-66.02 = \frac{-16(8100)}{v^2\cos^2 35}$

$-66 = \frac{-193142}{v^2}$

$v^2 = \frac{193142}{66}$

$v = 54.1$ ft/sec

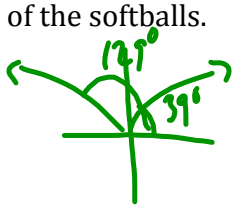
3. Use a graphing calculator to graph the path of the ball and its approximate maximum height.

4. Find the time the receiver has to position himself after the quarterback releases the football.

$t = \frac{90}{54.1\cos 35} = 2.03$ sec

1. Kelly and Mary are standing 78 feet apart.
At the same time, they each throw a softball toward each other.
Mary throws her softball with an initial velocity of 45 ft/sec with an angle of elevation of 44°.
Kelly throws her softball with an initial velocity of 41 ft/sec with an angle of elevation of 39°.

a) Write the sets of parametric equations to simulate the motion of the softballs.
b) Will the softballs collide? Justify your answer.
c) Which softball hits the ground first?
How far does each softball travel in the horizontal direction?



Mary

$x = 45t\cos 44$

$y = 45t\sin 44 - 16t^2$

Kelly

$x = 41t\cos 129 + 78$

$y = 41t\sin 129 - 16t^2$