

Product and Quotient Rules

Differentiate each function with respect to x .

1) $f(x) = (4\sqrt[5]{x} - 1) \cdot 5x^5$

2) $f(x) = \frac{1}{x^3}(-x^3 - 1)$

3) $f(x) = \left(3 + \frac{5}{x^3}\right) \cdot -2x^2$

4) $f(x) = (-1 - 4x^{-2}) \cdot 4x^5$

5) $f(x) = (x^2 - 4)(-x^5 + 5)$

6) $f(x) = (-2x^4 - 2)(-5x^4 - 5)$

7) $f(x) = (-x^3 - 5x^2 + 4)(-3x^2 + 2)$

8) $f(x) = (-5x^3 - 5)(2x^4 + 3x^2 - 5)$

$$9) f(x) = \frac{3x^3}{2 - x^{-4}}$$

$$10) f(x) = \frac{5x^5}{3 - \frac{3}{x^3}}$$

$$11) f(x) = \frac{1}{3 + 3x^{-2}}$$

$$12) f(x) = \frac{3}{5x^4 - 4}$$

$$13) f(x) = \frac{4x^2 - 4}{4x^2 - 5}$$

$$14) f(x) = \frac{2x^3 + 5x^2}{2x^4 + 4}$$

$$15) f(x) = \frac{5x^4 - 5x^3 - 2x^2}{4x^3 - 4}$$

$$16) f(x) = \frac{3x^4 + x^2 + 1}{3x^4 + 3}$$

Answers to Product and Quotient Rules

$$1) f'(x) = \left(4x^{\frac{1}{5}} - 1\right) \cdot 25x^4 + 5x^5 \cdot \frac{4}{5}x^{-\frac{4}{5}}$$

$$= 104x^{\frac{21}{5}} - 25x^4$$

$$3) f'(x) = (3 + 5x^{-3}) \cdot -4x - 2x^2 \cdot -15x^{-4}$$

$$= -12x + \frac{10}{x^2}$$

$$5) f'(x) = (x^2 - 4) \cdot -5x^4 + (-x^5 + 5) \cdot 2x$$

$$= -7x^6 + 20x^4 + 10x$$

$$7) f'(x) = (-x^3 - 5x^2 + 4) \cdot -6x + (-3x^2 + 2)(-3x^2 - 10x)$$

$$= 15x^4 + 60x^3 - 6x^2 - 44x$$

$$8) f'(x) = (-5x^3 - 5)(8x^3 + 6x) + (2x^4 + 3x^2 - 5) \cdot -15x^2$$

$$= -70x^6 - 75x^4 - 40x^3 + 75x^2 - 30x$$

$$9) f'(x) = \frac{(2 - x^{-4}) \cdot 9x^2 - 3x^3 \cdot 4x^{-5}}{(2 - x^{-4})^2}$$

$$= \frac{18x^{10} - 21x^6}{4x^8 - 4x^4 + 1}$$

$$11) f'(x) = -\frac{-6x^{-3}}{(3 + 3x^{-2})^2}$$

$$= \frac{2x}{3x^4 + 6x^2 + 3}$$

$$12) f'(x) = -\frac{3 \cdot \frac{25}{4}x^{\frac{1}{4}}}{\left(5x^{\frac{5}{4}} - 4\right)^2}$$

$$= -\frac{\frac{1}{75x^{\frac{1}{4}}}}{100x^{\frac{5}{2}} - 160x^{\frac{5}{4}} + 64}$$

$$13) f'(x) = \frac{(4x^2 - 5) \cdot 8x - (4x^2 - 4) \cdot 8x}{(4x^2 - 5)^2}$$

$$= -\frac{8x}{16x^4 - 40x^2 + 25}$$

$$14) f'(x) = \frac{(2x^4 + 4)(6x^2 + 10x) - (2x^3 + 5x^2) \cdot 8x^3}{(2x^4 + 4)^2}$$

$$= \frac{-x^6 - 5x^5 + 6x^2 + 10x}{x^8 + 4x^4 + 4}$$

$$15) f'(x) = \frac{(4x^3 - 4)(20x^3 - 15x^2 - 4x) - (5x^4 - 5x^3 - 2x^2) \cdot 12x^2}{(4x^3 - 4)^2}$$

$$= \frac{5x^6 + 2x^4 - 20x^3 + 15x^2 + 4x}{4x^6 - 8x^3 + 4}$$

$$16) f'(x) = \frac{(3x^4 + 3)(12x^3 + 2x) - (3x^4 + x^2 + 1) \cdot 12x^3}{(3x^4 + 3)^2}$$

$$= \frac{-2x^5 + 8x^3 + 2x}{3x^8 + 6x^4 + 3}$$

$$2) f'(x) = x^{-3} \cdot -3x^2 + (-x^3 - 1) \cdot -3x^{-4}$$

$$= \frac{3}{x^4}$$

$$4) f'(x) = (-1 - 4x^{-2}) \cdot 20x^4 + 4x^5 \cdot 8x^{-3}$$

$$= -20x^4 - 48x^2$$

$$6) f'(x) = (-2x^4 - 2) \cdot -20x^3 + (-5x^4 - 5) \cdot -8x^3$$

$$= 80x^7 + 80x^3$$

$$10) f'(x) = \frac{(3 - 3x^{-3}) \cdot 25x^4 - 5x^5 \cdot 9x^{-4}}{(3 - 3x^{-3})^2}$$

$$= \frac{25x^{10} - 40x^7}{3x^6 - 6x^3 + 3}$$