The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

$$\boxed{\frac{x^{n+1}}{n+1}}$$

Evaluating a Definite Integral

$$\int_1^2 (x^2 - 3)dx$$

$$= \frac{x^3}{3} - 3x \Big|_1^2 = \frac{2^3}{3} - 3(2) - \left(\frac{1^3}{3} - 3(1)\right)$$

$$\frac{7}{3} - 3 = -\frac{2}{3}$$

$$\int_1^4 3\sqrt{x}dx = \frac{3x^{3/2}}{3/2}\Big|_1^4 = 2x^{3/2}\Big|_1^4$$

$$= 2(4)^{3/2} - 2(1)^{3/2}$$

$$= 2(8) - 2(1) = 16 - 2 = 14$$

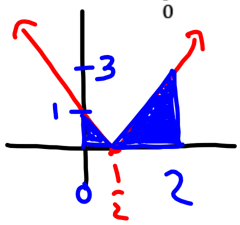
$$\int_0^{\pi/4} \sec^2 x\, dx$$

$$= \tan x + c \Big|_0^{\frac{\pi}{4}}$$

$$= \tan\left(\frac{\pi}{4}\right) + \cancel{c} - \left(\tan 0 + \cancel{c}\right)$$

$$= 1 - 0 = 1$$

$$\int_0^2 |2x - 1|dx$$



$$\int_0^{1/2} (-2x+1)dx + \int_{1/2}^2 (2x-1)dx$$

$$= -\frac{\cancel{2}x^2}{\cancel{2}} + x \Big|_0^{1/2} + \frac{\cancel{2}x^2}{\cancel{2}} - x \Big|_{1/2}^2$$

$$= -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + \left[2^2 - 2 - \left(\left(\frac{1}{2}\right)^2 - \frac{1}{2}\right)\right]$$

$$= -\frac{1}{4} + \frac{1}{2} + 4 - 2 - \frac{1}{4} + \frac{1}{2}$$

$$3 - \frac{2}{4} = 3 - \frac{1}{2} = \frac{5}{2} = 2.5$$

Finding Area

Find the area of the region bounded by the graph of $y = 2x^2 - 3x + 2$, the x-axis, and the vertical lines $x = 0$ and $x = 2$.

$$\int_0^2 (2x^2 - 3x + 2)\,dx = \left.\frac{2x^3}{3} - \frac{3x^2}{2} + 2x\,\right|_0^2$$

$$= \frac{2(2)^3}{3} - \frac{3(2)^2}{2} + 2(2)$$

$$= \frac{16}{3} - \frac{12}{2} + 4 = \frac{16}{3} - 6 + 4 = \frac{16}{3} - 2 = \frac{10}{3}$$

Mean Value Theorem for Integrals

Somewhere between the inscribed and circumscribed rectangles there is a rectangle whose area is precisely equal to the area of the region under the curve

$$\int_a^b f(x)dx = f(c)(b-a)$$

where c is some number that exists in the closed interval of $[a, b]$

Average Value of a Function

The value of $f(c)$ in the Mean Value Theorem for Integrals is called the average value

$$\frac{1}{b-a}\int_a^b f(x)dx$$

Find the average value of $f(x) = 3x^2 - 2x$ on the interval $[1, 4]$

$$\frac{1}{4-1}\int_1^4 (3x^2-2x)dx = \frac{1}{3}\left[\frac{\cancel{3}x^3}{\cancel{3}} - \frac{\cancel{2}x^2}{\cancel{2}}\right]_1^4 = \frac{1}{3}\left(4^3-4^2-\cancel{(1^3-1^2)}\right)$$

$$= \frac{1}{3}[64-16] = \frac{1}{3}(48) = 16$$