Calculus

Upper and Lower Sums

For each problem, find the area under the curve over the given interval. Set up your solution using the limit as *n* goes to ∞ of the lower sum.

1) $f(x) = x$; [0, 2]

2) $f(x) = 3x$; [0, 4]

3) $f(x) = 2x + 3$; [0, 5]

4) $f(x) = x + 5$; [0, 8]

For each problem, find the area under the curve over the given interval. Set up your solution using the limit as *n* goes to ∞ of the upper sum.

5) $f(x) = 2x$; [0, 4]

6) $f(x) = x^2$; [0, 1]

7) $f(x) = 2x^2 + 2$; [0, 2]

8) $f(x) = 2x + 2$; [0, 3]

For each problem, find the area under the curve over the given interval.

9) $f(x) = x^2$; [0, 2]

10) $f(x) = 2x^2$; [0, 1]

11) $f(x) = 2x$; [0, 6]

12) $f(x) = x^2 + 5$; [0, 1]

13) $f(x) = 3x + 2$; [0, 1]

14) $f(x) = x^2 + 1$; [0, 3]

Answers to Upper and Lower Sums

1) $$\lim_{n\to\infty} \sum_{k=1}^{n} \left(0 + \frac{2-0}{n}(k-1)\right) \cdot \frac{2-0}{n}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{4k}{n^2} - \frac{4}{n^2}\right)$$

$$\lim_{n\to\infty} \left(\frac{4}{n^2} \cdot \sum_{k=1}^{n} k - \frac{4}{n^2} \cdot \sum_{k=1}^{n} 1\right)$$

$$\lim_{n\to\infty} \left(\frac{4}{n^2} \cdot \frac{n(n+1)}{2} - \frac{4}{n^2} \cdot n\right)$$

$$\lim_{n\to\infty} \left(2 - \frac{2}{n}\right)$$

2

2) $$\lim_{n\to\infty} \sum_{k=1}^{n} 3\left(0 + \frac{4-0}{n}(k-1)\right) \cdot \frac{4-0}{n}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{48k}{n^2} - \frac{48}{n^2}\right)$$

$$\lim_{n\to\infty} \left(\frac{48}{n^2} \cdot \sum_{k=1}^{n} k - \frac{48}{n^2} \cdot \sum_{k=1}^{n} 1\right)$$

$$\lim_{n\to\infty} \left(\frac{48}{n^2} \cdot \frac{n(n+1)}{2} - \frac{48}{n^2} \cdot n\right)$$

$$\lim_{n\to\infty} \left(24 - \frac{24}{n}\right)$$

24

3) $$\lim_{n\to\infty} \sum_{k=1}^{n} \left(2\left(0 + \frac{5-0}{n}(k-1)\right) + 3\right) \cdot \frac{5-0}{n}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{15}{n} + \frac{50k}{n^2} - \frac{50}{n^2}\right)$$

$$\lim_{n\to\infty} \left(\frac{15}{n} \cdot \sum_{k=1}^{n} 1 + \frac{50}{n^2} \cdot \sum_{k=1}^{n} k - \frac{50}{n^2} \cdot \sum_{k=1}^{n} 1\right)$$

$$\lim_{n\to\infty} \left(\frac{15}{n} \cdot n + \frac{50}{n^2} \cdot \frac{n(n+1)}{2} - \frac{50}{n^2} \cdot n\right)$$

$$\lim_{n\to\infty} \left(40 - \frac{25}{n}\right)$$

40

4) $$\lim_{n\to\infty} \sum_{k=1}^{n} \left(0 + \frac{8-0}{n}(k-1) + 5\right) \cdot \frac{8-0}{n}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{40}{n} + \frac{64k}{n^2} - \frac{64}{n^2}\right)$$

$$\lim_{n\to\infty} \left(\frac{40}{n} \cdot \sum_{k=1}^{n} 1 + \frac{64}{n^2} \cdot \sum_{k=1}^{n} k - \frac{64}{n^2} \cdot \sum_{k=1}^{n} 1\right)$$

$$\lim_{n\to\infty} \left(\frac{40}{n} \cdot n + \frac{64}{n^2} \cdot \frac{n(n+1)}{2} - \frac{64}{n^2} \cdot n\right)$$

$$\lim_{n\to\infty} \left(72 - \frac{32}{n}\right)$$

72

5) $$\lim_{n\to\infty} \sum_{k=1}^{n} 2\left(0 + \frac{4-0}{n} \cdot k\right) \cdot \frac{4-0}{n}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{32k}{n^2}$$

$$\lim_{n\to\infty} \frac{32}{n^2} \cdot \sum_{k=1}^{n} k$$

$$\lim_{n\to\infty} \frac{32}{n^2} \cdot \frac{n(n+1)}{2}$$

$$\lim_{n\to\infty} \left(16 + \frac{16}{n}\right)$$

16

6) $$\lim_{n\to\infty} \sum_{k=1}^{n} \left(0 + \frac{1-0}{n} \cdot k\right)^2 \cdot \frac{1-0}{n}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^2}{n^3}$$

$$\lim_{n\to\infty} \frac{1}{n^3} \cdot \sum_{k=1}^{n} k^2$$

$$\lim_{n\to\infty} \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$\lim_{n\to\infty} \left(\frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right)$$

$\frac{1}{3} \approx 0.333$

7) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2\left(0 + \frac{2-0}{n}\cdot k\right)^2 + 2\right)\cdot\frac{2-0}{n}$

$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{4}{n} + \frac{16k^2}{n^3}\right)$

$\lim_{n\to\infty} \left(\frac{4}{n}\cdot\sum_{k=1}^{n} 1 + \frac{16}{n^3}\cdot\sum_{k=1}^{n} k^2\right)$

$\lim_{n\to\infty} \left(\frac{4}{n}\cdot n + \frac{16}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right)$

$\lim_{n\to\infty} \left(\frac{28}{3} + \frac{8}{n} + \frac{8}{3n^2}\right)$

$\frac{28}{3} \approx 9.333$

8) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2\left(0 + \frac{3-0}{n}\cdot k\right) + 2\right)\cdot\frac{3-0}{n}$

$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{6}{n} + \frac{18k}{n^2}\right)$

$\lim_{n\to\infty} \left(\frac{6}{n}\cdot\sum_{k=1}^{n} 1 + \frac{18}{n^2}\cdot\sum_{k=1}^{n} k\right)$

$\lim_{n\to\infty} \left(\frac{6}{n}\cdot n + \frac{18}{n^2}\cdot\frac{n(n+1)}{2}\right)$

$\lim_{n\to\infty} \left(15 + \frac{9}{n}\right)$

15

9) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(0 + \frac{2-0}{n}\cdot k\right)^2\cdot\frac{2-0}{n}$

$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{8k^2}{n^3}$

$\lim_{n\to\infty} \frac{8}{n^3}\cdot\sum_{k=1}^{n} k^2$

$\lim_{n\to\infty} \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$

$\lim_{n\to\infty} \left(\frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2}\right)$

$\frac{8}{3} \approx 2.667$

10) $\lim_{n\to\infty} \sum_{k=1}^{n} 2\left(0 + \frac{1-0}{n}\cdot k\right)^2\cdot\frac{1-0}{n}$

$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2k^2}{n^3}$

$\lim_{n\to\infty} \frac{2}{n^3}\cdot\sum_{k=1}^{n} k^2$

$\lim_{n\to\infty} \frac{2}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$

$\lim_{n\to\infty} \left(\frac{2}{3} + \frac{1}{n} + \frac{1}{3n^2}\right)$

$\frac{2}{3} \approx 0.667$

11) $\lim_{n\to\infty} \sum_{k=1}^{n} 2\left(0 + \frac{6-0}{n}\cdot k\right)\cdot\frac{6-0}{n}$

$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{72k}{n^2}$

$\lim_{n\to\infty} \frac{72}{n^2}\cdot\sum_{k=1}^{n} k$

$\lim_{n\to\infty} \frac{72}{n^2}\cdot\frac{n(n+1)}{2}$

$\lim_{n\to\infty} \left(36 + \frac{36}{n}\right)$

36

12) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\left(0 + \frac{1-0}{n}\cdot k\right)^2 + 5\right)\cdot\frac{1-0}{n}$

$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{5}{n} + \frac{k^2}{n^3}\right)$

$\lim_{n\to\infty} \left(\frac{5}{n}\cdot\sum_{k=1}^{n} 1 + \frac{1}{n^3}\cdot\sum_{k=1}^{n} k^2\right)$

$\lim_{n\to\infty} \left(\frac{5}{n}\cdot n + \frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right)$

$\lim_{n\to\infty} \left(\frac{16}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right)$

$\frac{16}{3} \approx 5.333$

13) $\lim_{n \to \infty} \sum_{k=1}^{n} \left(3\left(0 + \frac{1-0}{n} \cdot k\right) + 2\right) \cdot \frac{1-0}{n}$

$\lim_{n \to \infty} \sum_{k=1}^{n} \left(\frac{2}{n} + \frac{3k}{n^2}\right)$

$\lim_{n \to \infty} \left(\frac{2}{n} \cdot \sum_{k=1}^{n} 1 + \frac{3}{n^2} \cdot \sum_{k=1}^{n} k\right)$

$\lim_{n \to \infty} \left(\frac{2}{n} \cdot n + \frac{3}{n^2} \cdot \frac{n(n+1)}{2}\right)$

$\lim_{n \to \infty} \left(\frac{7}{2} + \frac{3}{2n}\right)$

$\frac{7}{2} = 3.5$

14) $\lim_{n \to \infty} \sum_{k=1}^{n} \left(\left(0 + \frac{3-0}{n} \cdot k\right)^2 + 1\right) \cdot \frac{3-0}{n}$

$\lim_{n \to \infty} \sum_{k=1}^{n} \left(\frac{3}{n} + \frac{27k^2}{n^3}\right)$

$\lim_{n \to \infty} \left(\frac{3}{n} \cdot \sum_{k=1}^{n} 1 + \frac{27}{n^3} \cdot \sum_{k=1}^{n} k^2\right)$

$\lim_{n \to \infty} \left(\frac{3}{n} \cdot n + \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}\right)$

$\lim_{n \to \infty} \left(12 + \frac{27}{2n} + \frac{9}{2n^2}\right)$

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