

Volume

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the the x -axis.

1) $y = \csc x$, $y = 0$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{2}$

2) $y = \sqrt{x}$, $y = 0$, $x = 1$

3) $y = -x^2 + 1$, $y = 0$, $x = 0$, $x = 1$

4) $y = x^2$, $y = 0$, $x = 2$

5) $y = \sqrt{x}$, $y = 0$, $x = 4$

6) $y = \sec x$, $y = 0$, $x = -\frac{\pi}{3}$, $x = \frac{\pi}{4}$

7) $y = \csc x$, $y = 0$, $x = \frac{\pi}{3}$, $x = \frac{2\pi}{3}$

8) $y = -x^2 + 4$, $y = 0$

$$9) y = \csc x, y = 0, x = \frac{\pi}{2}, x = \frac{2\pi}{3}$$

$$10) y = \sqrt[3]{x}, y = 0, x = 1$$

Answers to Volume

$$1) \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \csc^2 x \, dx$$

$$= \sqrt{3} \cdot \pi \approx 5.441$$

$$5) \pi \int_0^4 (\sqrt{x})^2 \, dx$$

$$= 8\pi \approx 25.133$$

$$8) \pi \int_{-2}^2 (-x^2 + 4)^2 \, dx$$

$$= \frac{512}{15}\pi \approx 107.233$$

$$2) \pi \int_0^1 (\sqrt{x})^2 \, dx$$

$$= \frac{1}{2}\pi \approx 1.571$$

$$6) \pi \int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} \sec^2 x \, dx$$

$$= (1 + \sqrt{3})\pi \approx 8.583$$

$$9) \pi \int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \csc^2 x \, dx$$

$$= \frac{\sqrt{3}}{3}\pi \approx 1.814$$

$$3) \pi \int_0^1 (-x^2 + 1)^2 \, dx$$

$$= \frac{8}{15}\pi \approx 1.676$$

$$7) \pi \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \csc^2 x \, dx$$

$$= \frac{2\sqrt{3}}{3}\pi \approx 3.628$$

$$10) \pi \int_0^1 (\sqrt[3]{x})^2 \, dx$$

$$= \frac{3}{5}\pi \approx 1.885$$

$$4) \pi \int_0^2 (x^2)^2 \, dx$$

$$= \frac{32}{5}\pi \approx 20.106$$